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Optimal Design of Batch-Storage Network Considering Exchange Rates and Taxes

Gyeongbeom Yi
Dept. of Chemical Engineering, Pukyong National University, Nam-Ku, Busan, Korea 608-739

Gintaras V. Reklaitis
School of Chemical Engineering, Purdue University, West Lafayette, IN 47907

DOI 10.1002/aic.11161
Published online April 4, 2007 in Wiley InterScience (www.interscience.wiley.com).

This article presents an integrated analysis of the supply chain and financing decisions of multinational corporations. We construct a model in which multiple currency storage units are installed to manage the currency flows associated with multinational supply chain activities such as raw material procurement, processing, inventory control, transportation, and finished product sales. Temporary financial investments, bank loans, and currency transfer between multiple nations are allowed to increase the marginal profit. The core contribution of this study is its quantitative investigation of the influence of macroscopic economic factors such as exchange rates and taxes on operational decisions. The supply chain is modeled as a batch-storage network with recycling streams. The objective function of the optimization involves minimizing the opportunity costs of annualized capital investments and currency/material inventories minus the benefit to stockholders in the numeraire currency. The major constraints of the optimization are that the material and currency storage units must not be depleted. A production and inventory analysis formulation (the periodic square wave model) provides useful expressions for the upper and lower bounds and average levels of the currency and material inventory holdings. The expressions for the Kuhn-Tucker conditions of the optimization problem are reduced to a subproblem and analytical lot-sizing equations. The lot sizes of procurement, production, transportation, and financial transaction can be determined by analytical expressions once the average flow rates are known. We show that the optimal production lot and storage sizes are typically 20% smaller when corporate income tax is taken into consideration than when it is not considered. An illustrative example is presented to demonstrate the potential of this approach. *AIChE J,* 53: 1211–1231, 2007
Keywords: optimal lot-size, batch-storage network, multicurrency, multinational corporation, exchange rate, corporate income tax, customs duty

Introduction

Major chemical industries are currently experiencing multiple difficulties such as lower customer demand, lower margins, and fierce competition. In order for these industries to stay profitable, they aggressively merge similar enterprises in order to take advantage of the economies of scale not only from slimming the manpower of business management and/or processing operations, but also from consolidating production into bigger plants with higher efficiency. Such changes are taking place on global basis and hence multinational corporations (MNCs) are now very common in the chemical

Correspondence concerning this article should be addressed to G. Yi at gbyi@pknu.ac.kr.

industry. An MNC purchases raw materials from many nations, produces products from plants in many (possibly different) nations and then sells the products all over the world. Moreover, the MNC hires employees from many nations and typically pays their salaries in their own currencies. Therefore, an MNC needs to cope with multiple currencies when managing its multinational business, which exposes the corporation to foreign currency exchange risks associated with the values of currencies fluctuating in accordance with the international economic environment. For example, an MNC might lose its profit without doing anything if it possesses an excessive amount of a currency whose value depreciates. In these uncertain times, variations in the exchange rate between two currencies can be quite dramatic. For example, the yen appreciated relative to the US dollar by 10% annually from the beginning of 2002 to the end of 2004. Under these circumstances, a manufacturer in Japan may have had to sell a product to the USA at below the production cost if the selling price had been fixed at the original exchange rate. Many financial techniques are used to mitigate this risk, such as options, forwards and the swapping of currencies and/or exchange rates. An MNC should therefore pay special attention to predicting variations in exchange rates, and conduct appropriate financial activities (which are not the subject of this study). A change in the exchange rate influences not only the financial activities of an MNC but also the production activities and supply chain management. When such a corporation possesses production plants in two nations, one with a strong currency and one with a weak currency, it should minimize the production volume of plants in the former nation and maximize that in the latter. Moreover, the MNC should purchase raw materials from suppliers dealing in a relatively weak currency. The optimization model of production planning and scheduling differs when more than two currencies are involved, and consequently the optimal solution will also be different, which is the motivation of this study. The plants in different nations have diverse input data of flows, stocks, timings, estimations, costs, benefits, and prices, all in different currencies. The global optimal solution of an MNC is strongly influenced by the handling of multiple currencies among such input data collected from multiple nations.

Another macroscopic economic factor that is important for MNCs is differences in tax rates between nations. Among the many kinds of taxes, sales income tax, corporate income tax and customs duty are especially important to MNCs. A previous study[1] found that sale tax did not influence any lot-sizing decisions or cost factors, but that it could influence the currency balance and therefore the timing of currency flows. The importance of corporate income tax to the feasibility of a new business is emphasized in a well-known plant design textbook.[2] However, many researchers still select the gross profit before tax as the performance measure to maximize in their production planning and scheduling optimization. The maximum corporate income tax rates currently range from 12.5% (in Ireland) to 40.8% (in the USA), with the average rate in OECD member countries being 30.8%. There is no doubt that the tax policies of the relevant nations should be investigated thoroughly in a real business, but surprisingly, the impact of corporate income tax on business optimality is not fully elucidated in the open research literature. This is because most research work has employed linear models, which provide the same solutions for the objective functions of the profits before and after tax as far as the relevant constraints are not binding. When nonlinear factors are included in the model, the optimality can differ depending on whether or not corporate income tax is considered.[3] Moreover, customs duty is an important cost factor that influences international business structures, with it being common for merchandise to pass through a third country in order to reduce customs duty.

Literature Review

This section describes a few recent operations management papers about exchange rates. A two-stage recourse model has been used to value the economics of production and allocate hedging under exchange rate uncertainty[4] where production hedging involves the firm deliberately producing less than the total demand, and allocation hedging means that some markets are not served despite having unused production. The effect of exchange rates on the long-term ownership strategies of production facilities of firms entering foreign markets under the presence of switch-over costs has also been investigated,[5] with the strategies considered including exporting, joint ventures with local partners, and wholly owned production facilities in foreign countries. A stochastic dynamic programming formulation for valuing global manufacturing strategy options with switch-over costs was developed,[6] where the global manufacturing strategy of an MNC determined options for alternative product designs as well as supply chain network designs. Exchange rates were modeled as stochastic diffusion processes that exhibited intercountry correlations. The operations research investigations have considered many factors that influence a multinational business, such as uncertainty in exchange rates, switch-over costs, and ownership, by applying the new methodology of option valuation theory. However, only linear mathematical models have been applied, with the variables aggregated to the level of the plant or nation. In contrast, the present study uses simple analytical solutions to determine optimal decisions at the level of processes and storage units located on arbitrary supply chain networks.

A novel production and inventory analysis method called the periodic square wave (PSW) method has been used to determine the optimal design of a parallel batch-storage system.[7] The PSW formalism was subsequently extended to model the more complicated plant structure of a sequential multistage batch-storage network (BSN).[8] In another studies, the same authors suggested a nonsequential network structure that can cope with recycled material flows,[9] semi-continuous processes[10] and financial transactions required to support the production activities.[1]

Problem Characteristics

This study focuses on quantifying the effect of exchange rate and taxes to the optimal design and operation of manufacturing MNCs. To the authors' knowledge, the introduction of exchange rate into the optimization model is unique in the chemical engineering literature. This study uses BSN to represent the global production plants and develops an optimization formulation using a PSW model. The case with a single currency and without corporate income tax and customs duty has already been studied.[1] The present study extends this to the case with multiple currencies and taxes. The uncertainty in exchange rates is a major research issue in operations research,[4–6] and analytical optimal lot-sizing solutions for short-term random variations and multiperiod formulations for catching up with slowly time-varying nature of material flows have already been developed.[11] Therefore, it should be possible to cope with all the important factors of exchange rates simultaneously, however, due to the complexity of the problem, including the uncertainty of exchange rates was postponed to future research. Tax policies differ greatly between nations but this study applies the further simplification of considering only linear tax rates, with the sales tax being proportional to sales volume, the corporate income tax being proportional to the gross profit before tax and the customs duty being proportional to the flow of material across a national border. Note that there are many types of exception to linear tax rates. For example, the corporate income tax rate can discontinuously increase according to the ranges of gross profit before tax,[2] and the customs duty can be drawn back when imported raw materials are used to produce exported goods.[12]

In the model proposed in this study, all production activities are accompanied by financial transactions in which the appropriate amount of currency is withdrawn from a currency storage unit to pay for the costs. The currency is inputted to the storage after delivery of the finished product to consumers. The currency inventory should be managed so as to ensure that it is not depleted. The objective function of the optimization involves minimizing the opportunity costs of annualized capital investment and currency/material inventory minus the benefit to stockholders in the numeraire currency. The unique features of the present study relative to previous studies are enlarging the BSN to cover multiple currencies and nations with the inclusion of the effects of exchange rates, taxes, depreciations, transfer prices, and bank loans.

The theory developed in this study is based on the following major assumptions:

(i) All production and financial operations are periodic.

(ii) The long-term material and currency flows are balanced.

(iii) The currency and material inventories can not be depleted.

(iv) Annualized depreciations are equal to annualized capital investments which are proportional to their capacities.

(v) The currency transfer cost is paid by the receiving subsidiary.

(vi) The currency flows of depreciations, dividends to stockholders and corporate income tax are continuous.

(vii) Corporate income tax is proportional to gross profit.

(viii) Customs duties are proportional to the material flows.

(ix) Corporate income tax and stockholder dividends begin to be paid at the same time.

(x) Salvage value is ignored.

(xi) There is no double taxation.

Many of these assumptions can be relaxed to cope with differing real conditions if necessary. Among the assumptions, the most unusual one is that the currency flows of depreciations, dividends to stockholders, and corporate income tax are continuous, whereas such currency flows usually happen once a year in MNCs. This assumption can be relaxed in a few ways, including by assuming that the currency transaction lot size is constant.[1]

In the next section, we define relevant variables and parameters for material and currency flows. The subsequent section formulates the optimization method and then the analytical solution of Kuhn-Tucker conditions is derived.

Definition of Variables and Parameters

This study applies a BSN to internationally spread production plants and logistic facilities of an MNC[9]; below we briefly define the variables and some of the equations used in the present study. A supply chain system, that converts raw materials into final products through multiple physicochemical processing steps and transports them to customers, is composed of a set of currency storage units (R), a set of material storage units (J) and a set of batch processes (I), as shown in Figure 1; note that semi-continuous processes can be included into model with additional mathematical treatment.[10] The definition of currency set R is very important, with the same currency in different nations being considered as different members of the set. For example, the US dollar units in the USA and Korea are different currency set members; that is, the exact meaning of set R is the "currency used in the nation." Therefore, since multiple currencies can be used within a given nation, the total number of members in set R is equal to the number of currencies multiplied by the number of nations. Of course, many nations use only one currency, and so superscript $r \in R$ represents a nation as well as a currency without loss of generality. Each process requires multiple feedstock materials of fixed composition (f_i^j) and produces multiple products with fixed product yield (g_i^j), as shown in Figure 2a. If there is no material flow between a storage unit and a process unit, the corresponding feedstock composition or product yield is zero. Each storage unit is filled with one material, but the same material can be stored in multiple storage units located at different sites. As shown in Figure 2b, each storage unit is associated with four types of material movement: purchasing from suppliers ($k \in K(j)$), shipping to consumers ($m \in M(j)$), feeding to processes and production from processes. Note that the sets of suppliers $K(j)$ and consumers $M(j)$ are storage dependent. The material flow from a process to a storage unit (or from a storage unit to a process) is represented by the PSW model, as shown in Figure 2c where the area beneath each block represents the amount of material produced by the process. In the PSW model, material flow is represented in terms of four variables: batch size B_i, cycle time ω_i, storage operation time fraction (SOTF) $'x_i$ (or x_i'), and startup time $'t_i$ (or t_i'). The SOTF $'x_i$ (or x_i') is defined as the time required for material movement to (or from) the process divided by the cycle time. The startup time $'t_i$ (or t_i') is the first time at which the first batch is fed into (or discharged from) the process. Assume that the operations feeding feedstock to the process (or the operations discharging product from the process) occur simultaneously and that their SOTFs are the same. That is, the superscript j is not necessary to discriminate the related storage units in $'x_i$ (or x_i') and $'t_i$ (or t_i'). The material flow of purchased raw material is represented by the order

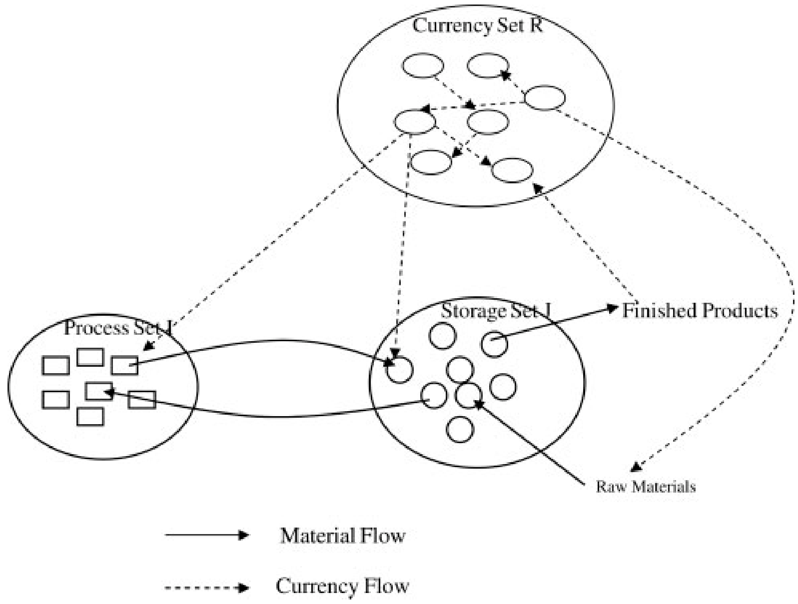

Figure 1. Basic structure of multi-national corporation.

size B_k^j, cycle time ω_k^j, SOTF x_k^j, and startup time t_k^j. All SOTFs are considered as parameters, whereas the other variables are the design variables used in this study. The material flow of finished product sales is represented by B_m^j, ω_m^j, x_m^j, and t_m^j in the same way. The arbitrary periodic function of forecasted demand for the finished product can be represented by a sum of PSW functions with known values of B_m^j, ω_m^j, x_m^j, and t_m^j.[7] The general form of PSW functions is defined as follows:

$$PSW(t; D, \omega, t', x) = D\omega\left[\text{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x} res\left[\frac{t-t'}{\omega}\right]\right\}\right] \quad (1)$$

or

$$PSW'(t; B, \omega, t', x) = B\left[\text{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x} res\left[\frac{t-t'}{\omega}\right]\right\}\right] \quad (2)$$

where D is the average flow rate, B is the batch size, ω is the cycle time, t' is the startup time, x is the SOTF, t is time, int[z] is the greatest integer less than or equal to z and res[z] = z-int[z]. Note that $D = \frac{B}{\omega}$ and PSW' (t; $g_i^j B_i$, ω_i, t_i', x_i') $= \int_0^t F_i(t)dt$ where $F_i(t)$ is given at Figure 2c. Equation 1 is referred to the first type of PSW flow and Equation 2 to the second type of PSW flow. Note that the average flow rate is used in the first type whereas the batch size is used in the second type. The two types of PSW flow have different upper and lower bounds and partial derivatives. Figure 3 shows the bounds on the second type of PSW function. Table 1 lists the expressions for the average and upper and lower bounds of the first and second types of PSW flows,[1] where $\underline{\underline{PSW}} \le PSW \le \overline{\overline{PSW}}$, $\underline{\underline{PSW'}} \le PSW' \le \overline{\overline{PSW'}}$, $\overline{PSW} = 0.5(\overline{\overline{PSW}} + \underline{\underline{PSW}})$, and $\overline{PSW'} = 0.5(\overline{\overline{PSW'}} + \underline{\underline{PSW'}})$.

Let D_i be the average material flow rate through process i, which can be expressed as batch size B_i divided by cycle time ω_i. The average material flows of raw material purchase from suppliers and finished product shipping to consumers are denoted by D_k^j and D_m^j, respectively, where $D_k^j = (B_k^j/\omega_k^j)$ and $D_m^j = (B_m^j/\omega_m^j)$. The initial inventory of storage j is denoted by $V^j(0)$ and the inventory held in storage j at time t is denoted by $V^j(t)$.

The purchasing setup cost of raw material j paid in currency r is denoted by A_k^{jr} (currency/order) and the setup cost of process i paid in currency r is denoted by A_i^r (currency/batch). The annual inventory holding cost of storage j paid in currency r is denoted by H^{jr} (currency/L/year). Note that all material flows are measured volumetrically for convenience. The inventory holding cost is further segregated into the inventory operating cost (h^{jr}) and the opportunity cost of inventory holding (γ^{jr}); that is, $H^{jr} = h^{jr} + \gamma^{jr}$. To obtain an analytical solution, the capital cost is assumed to be proportional to the processing capacity. Suppose that a_k^{jr} (currency/L/year) is the annual capital cost per unit capacity of the purchasing facility for raw material j paid in currency r, a_i^r (currency/L/year) is the annual capital cost per unit capacity of process i paid in currency r, and b^{jr} (currency/L/year) is the annual capital cost per unit capacity of storage j paid in currency r. In addition, assume that the raw material cost is proportional to the quantity and purchase price of raw material j from supplier k paid in currency r is P_k^{jr} (currency/L). The sales price of finished product j to consumer m paid in currency r is P_m^{jr} (currency/L).

Given that one production cycle in a process comprises the feedstock feeding time (${}'x_i\omega_i$), processing time ($(1 - {}'x_i - x_i')\omega_i$), and product discharging time ($x_i'\omega_i$), the timing relationship between the startup time of the feedstock streams and the startup time of the product streams is.

$$t_i' = {}'t_i + \Delta t_i\,(.) \qquad \forall i \quad (3)$$

The overall material balance associated with the material storage results in the following relationship:

$$\sum_{i=1}^{|I|} g_i^j D_i + \sum_{k=1}^{|K(j)|} D_k^j = \sum_{i=1}^{|I|} f_i^j D_i + \sum_{m=1}^{|M(j)|} D_m^j \qquad \forall j \quad (4)$$

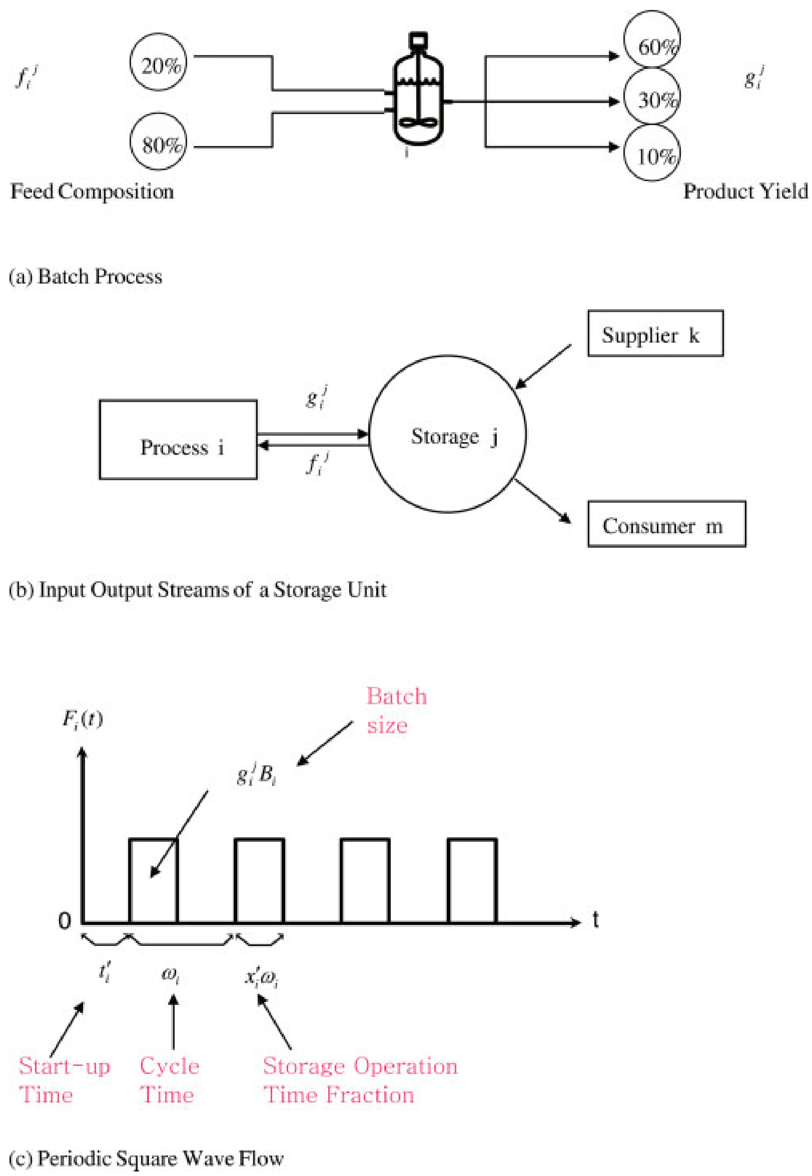

Figure 2. Structure of batch-storage network. (a) Batch process (b) Input output streams of a storage unit (c) Periodic square wave flow.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

A material storage unit is connected to the incoming flows from suppliers and processes and the outgoing flows to consumers and processes. The resulting inventory holding function for a material storage unit is:

$$V^j(t) = V^j(0) + \sum_{k=1}^{|K(j)|} PSW\left(t; D_k^j, \omega_k^j, t_k^j, x_k^j\right) + \sum_{i=1}^{|I|} PSW(t; g_i^j D_i, \omega_i, t_i', x_i') - \sum_{i=1}^{|I|} PSW\left(t; f_i^j D_i, \omega_i, {}'t_i, {}'x_i\right) - \sum_{m=1}^{|M(j)|} PSW(t; D_m^j, \omega_m^j, t_m^j, x_m^j) \qquad \forall j \quad (5)$$

The upper and lower bounds and the average value of the inventory holding are calculated using the properties of the PSW functions in Table 1.

$$\overline{\overline{V^j}} = V^j(0) + \sum_{k=1}^{|K(j)|} \overline{\overline{PSW}}(t; D_k^j, \omega_k^j, t_k^j, x_k^j) + \sum_{i=1}^{|I|} \overline{\overline{PSW}}(t; g_i^j D_i, \omega_i, t_i', x_i') - \sum_{i=1}^{|I|} \underline{\underline{PSW}}(t; f_i^j D_i, \omega_i, {}'t_i, {}'x_i) - \sum_{m=1}^{|M(j)|} \underline{\underline{PSW}}(t; D_m^j, \omega_m^j, t_m^j, x_m^j)$$
$$= V^j(0) + \sum_{k=1}^{|K(j)|} (1 - x_k^j) D_k^j \omega_k^j - \sum_{k=1}^{|K(j)|} D_k^j t_k^j + \sum_{i=1}^{|I|} (1 - x_i') g_i^j D_i \omega_i - \sum_{i=1}^{|I|} g_i^j D_i t_i' + \sum_{i=1}^{|I|} f_i^j D_i {}'t_i + \sum_{m=1}^{|M(j)|} D_m^j t_m^j \qquad \forall j \quad (6)$$

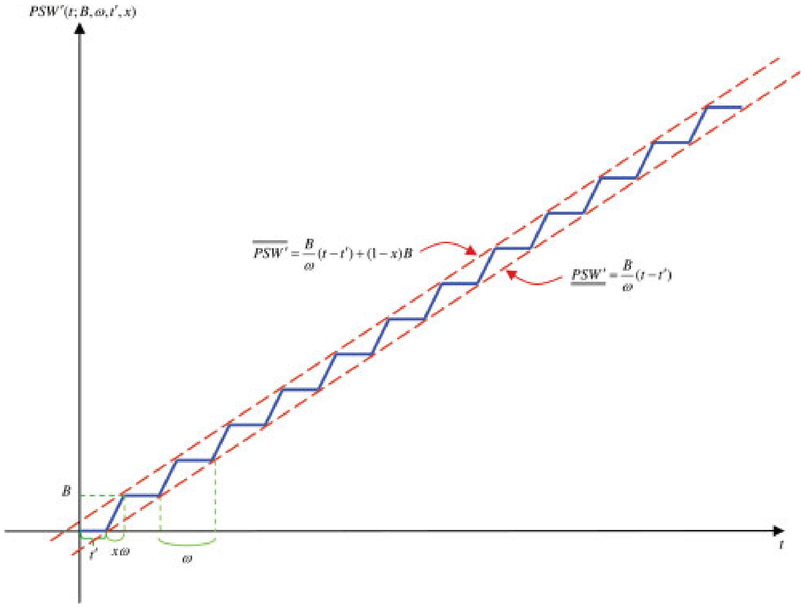

Figure 3. Bounds on PSW Function.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

$$\underline{\underline{V^j}} = V^j(0) + \sum_{k=1}^{|K(j)|} \underline{\underline{PSW}}(t; D_k^j, \omega_k^j, t_k^j, x_k^j) + \sum_{i=1}^{|I|} \underline{\underline{PSW}}(t; g_i^j D_i, \omega_i, t_i', x_i')$$
$$- \sum_{i=1}^{|I|} \overline{\overline{PSW}}(t; f_i^j D_i, \omega_i, {}'t_i, {}'x_i) - \sum_{m=1}^{|M(j)|} \overline{\overline{PSW}}(t; D_m^j, \omega_m^j, t_m^j, x_m^j)$$
$$= V^j(0) - \sum_{k=1}^{|K(j)|} D_k^j t_k^j - \sum_{i=1}^{|I|} g_i^j D_i t_i' - \sum_{i=1}^{|I|} (1 - {}'x_i) f_i^j D_i \omega_i$$
$$+ \sum_{i=1}^{|I|} f_i^j D_i {}'t_i - \sum_{m=1}^{|M(j)|} (1 - x_m^j) D_m^j \omega_m^j + \sum_{m=1}^{|M(j)|} D_m^j t_m^j \quad \forall j \quad (7)$$

$$\overline{V^j} = V^j(0) + \sum_{k=1}^{|K(j)|} \overline{PSW}(t; D_k^j, \omega_k^j, t_k^j, x_k^j) + \sum_{i=1}^{|I|} \overline{PSW}(t; g_i^j D_i, \omega_i, t_i', x_i')$$
$$- \sum_{i=1}^{|I|} \overline{PSW}(t; f_i^j D_i, \omega_i, {}'t_i, {}'x_i) - \sum_{m=1}^{|M(j)|} \overline{PSW}(t; D_m^j, \omega_m^j, t_m^j, x_m^j)$$
$$= V^j(0) + \sum_{k=1}^{|K(j)|} \frac{(1 - x_k^j)}{2} D_k^j \omega_k^j - \sum_{k=1}^{|K(j)|} D_k^j t_k^j + \sum_{i=1}^{|I|} \frac{(1 - x_i')}{2} g_i^j D_i \omega_i$$
$$- \sum_{i=1}^{|I|} g_i^j D_i t_i' - \sum_{i=1}^{|I|} \frac{(1 - {}'x_i)}{2} f_i^j D_i \omega_i + \sum_{i-1}^{|I|} f_i^j D_i {}'t_i$$
$$- \sum_{m=1}^{|M(j)|} \frac{(1 - x_m^j)}{2} D_m^j \omega_m^j + \sum_{m=1}^{|M(j)|} D_m^j t_m^j \quad \forall j \quad (8)$$

Equation 6 is used to predict the storage size, Eq. 7 is used to implement the no-depletion constraint, and Eq. 8 is used to calculate the inventory holding cost.

Suppose that there exist currency storage units of the type shown in Figure 4 that, through financial transactions, operate the supply chain composed of batch process set I and material storage unit set J, as depicted in Figure 1. Let set N with subscript n represent the set of temporary financial investments in marketable securities, set L with subscript l represent the set of bank loans and set O with subscript o represent the set of stockholders. Corporate income tax is usually proportional to gross profit and thus, without loss of generality, is considered as a payment to a fictitious stockholder $\bar{o} \in O$. Sales tax, which is usually proportional to sales revenue, is collected from customers when finished products are delivered to them and is paid to the IRS (Internal Revenue Service) yearly. In chemical companies, the total labor cost is usually proportional to the total sales revenue. We ignore the currency flow of labor costs in the present study because it is treated in the same way as sales tax. Note that the setup cost usually includes the operating labor cost. The currency flows entering the currency storage unit are as follows:

(CF1) Collection of account receivable after collection drifting time Δt_m^j (year) from shipping of the finished product to consumer m (including sales tax).

(CF2) Return from temporary financial investment n at interest rate κ_n^r (currency/currency/year) after investment period Δt_n^r (year).

Table 1. Mathematical Properties of PSW Flows

Average of first type PSW flow	$\overline{PSW}(t; D, \omega, t', x) = D(t - t') + 0.5(1 - x)D\omega$
Average of second type PSW flow	$\overline{PSW'}(t; B, \omega, t', x) = \frac{B}{\omega}(t - t') + 0.5(1 - x)B$
Upper bound of first type PSW flow	$\overline{\overline{PSW}}(t; D, \omega, t', x) = D(t - t') + (1 - x)D\omega$
Upper bound of second type PSW flow	$\overline{\overline{PSW'}}(t; B, \omega, t', x) = \frac{B}{\omega}(t - t') + (1 - x)B$
Lower bound of first type PSW flow	$\underline{\underline{PSW}}(t; D, \omega, t', x) = D(t - t')$
Lower bound of second type PSW flow	$\underline{\underline{PSW'}}(t; B, \omega, t', x) = \frac{B}{\omega}(t - t')$

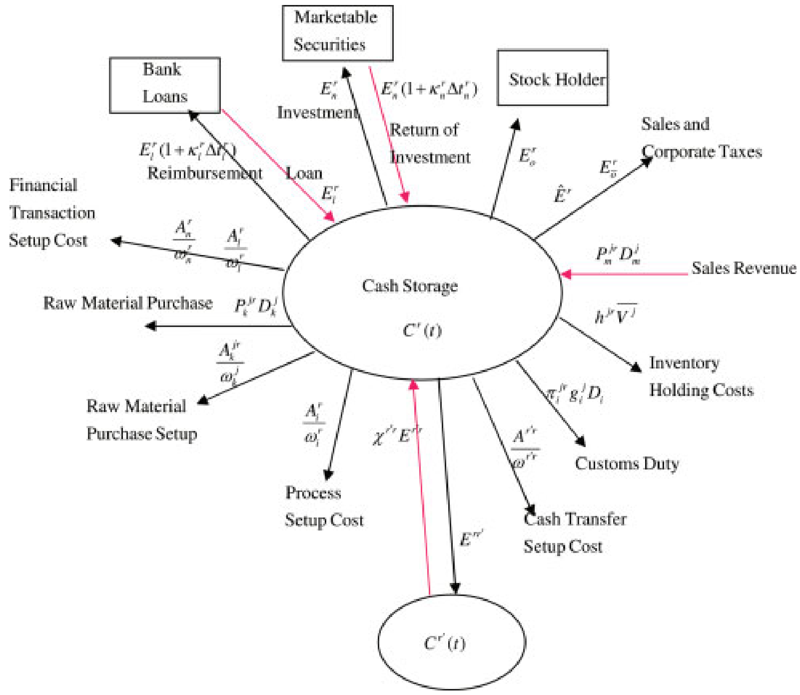

Figure 4. Currency storage and financial transactions.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

(CF3) Bank loans at interest rate κ_l^r (currency/currency/year) for loan period Δt_l^r (year).

(CF4) Currency transfer from currency storage r' to currency storage r with exchange rate $\chi^{r'r}$ (currency/currency).

The currency flows leaving the currency storage unit are as follows:

(CF5) Disbursement of account payable after disbursement drifting time Δt_k^j (year) for a raw material purchase from supplier k.

(CF6) Temporary financial investment at interest rate κ_n^r for investment period Δt_n^r.

(CF7) Reimbursement of bank loans l at interest rate κ_l^r after loan period Δt_l^r.

(CF8) Currency transfer from currency storage r to currency storage r' with exchange rate $\chi^{rr'}$.

(CF9) Purchase setup cost A_k^{jr} (currency/transaction).

(CF10) Processing setup cost A_i^r (currency/transaction).

(CF11) Investment setup cost A_n^r (currency/transaction).

(CF12) Bank loan setup cost A_l^r (currency/transaction).

(CF13) Incoming currency transfer setup cost $A^{r'r}$ (currency/transaction).

(CF14) Inventory operating cost h^{jr}.

(CF15) Dividend to stockholders, which includes the corporate income tax paid in currency r to the IRS at a rate ξ^r (currency/currency).

(CF16) Sales income tax paid in currency r to the IRS at a rate ς^r (currency/currency). (The labor cost can be treated as the same way.)

(CF17) Customs duty payment proportional to the material flow rate at a rate π_i^{jr} (currency/L). (A variable operating cost can be treated as the same way.)

In the present work we do not consider the case of an MNC taking bank loans to top up their initial currency reserves because, once any initial currency shortage has been addressed, it should be unnecessary to take further loans. The bank loans in CF3 and CF7 are for the strategic purpose of taking advantage of an unusually low interest rate in a certain nation; these bank loans are periodically repeated. Assume that the bank loan has a setup cost of A_l^r (currency/transaction) paid in currency r. This transaction cost is withdrawn from the currency storage when the bank loan is made, as is defined in CF12. The currency flows of financial investment, (CF2, CF6, and CF11), are defined as the same way with a transaction setup cost of A_n^r (currency/transaction). The currency transfers between currency storage units, (CF4, CF8, and CF13). We denote the average currency flow rate $E^{r'r}$ (currency/year) from currency storage r' to currency storage r with exchange rate $\chi^{r'r}$. The corresponding transfer setup cost $A^{r'r}$ (currency/transaction) is paid by receiving currency storage r. In addition, we assume that the setup cost transactions of CF9–CF13 and the inventory operating cost of CF14 are paid on a pro rata basis according to the progress of the material processing or financial transaction volume. In other words, the currency flow of the setup cost transaction and its material or currency flow have the same cycle time, startup time, and SOTF, but different batch size. The currency flow of the inventory operating cost is proportional to the inventory level. Each currency flow is represented in the PSW model with variables or parameters of batch size (or average flow rate), cycle time, startup time, and SOTF in the same way as material flows (with appropriate super- or subscripts).

The average flow rate of sales tax is proportional to that of total sales revenue; that is, $\hat{E}^r = \varsigma^r \sum_{j=1}^{|J|} \sum_{m=1}^{|M(j)|} P_m^{jr} D_m^j$, where ς^r (currency/currency) is the sales tax rate of the nation that uses currency r. The cycle time, startup time, and

Table 2. Mathematical Representation of Currency Flows

	PSW flows
CF1	$\sum_{j=1}^{\lvert J\rvert}\sum_{m=1}^{\lvert M(j)\rvert}(1+\varsigma^r)P_m^{jr}PSW(t;D_m^j,\omega_m^j,t_m^j+\Delta t_m^j,x_m^j)$
CF2	$\sum_{n=1}^{\lvert N\rvert}PSW(t;E_n^r(1+\kappa_n^r\Delta t_n^r),\omega_n^r,t_n^r+\Delta t_n^r,0)$
CF3	$\sum_{l=1}^{\lvert L\rvert}PSW(t;E_l^r,\omega_l^r,t_l^r,0)$
CF4	$\sum_{r'\neq r}^{\lvert R\rvert}\chi^{r'r}PSW(t;E^{r'r},\omega^{r'r},t^{r'r},0)$
CF5	$\sum_{j=1}^{\lvert J\rvert}\sum_{k=1}^{\lvert K(j)\rvert}P_k^{jr}PSW(t;D_k^j,\omega_k^j,t_k^j+\Delta t_k^j,x_k^j)$
CF6	$\sum_{n=1}^{\lvert N\rvert}PSW(t;E_n^r,\omega_n^r,t_n^r,0)$
CF7	$\sum_{n=1}^{\lvert N\rvert}PSW(t;E_l^r(1+\kappa_l^r\Delta t_l^r),\omega_l^r,t_l^r+\Delta t_l^r,0)$
CF8	$\sum_{r'\neq r}^{\lvert R\rvert}PSW(t;E^{rr'},\omega^{rr'},t^{rr'},0)$
CF9	$\sum_{j=1}^{\lvert J\rvert}\sum_{k\in\{D_k^j\}^+}^{\lvert K(j)\rvert}PSW'(t;A_k^{jr},\omega_k^j,t_k^j,x_k^j)$
CF10	$\sum_{i\in\{D_i\}^+}^{\lvert I\rvert}PSW'(t;A_i^r,\omega_i,'t_i,'x_i)$
CF11	$\sum_{n\in\{E_n^r\}^+}^{\lvert N\rvert}PSW'(t;A_n^r,\omega_n^r,t_n^r,0)$
CF12	$\sum_{l\in\{E_l^r\}^+}^{\lvert L\rvert}PSW'(t;A_l^r,\omega_l^r,t_l^r,0)$
CF13	$\sum_{r,r'\in\{E^{r'r}\}^+}^{\lvert R\rvert}PSW(t;A^{r'r},\omega^{r'r},t^{r'r},0)$
CF14	$\sum_{j=1}^{\lvert J\rvert}h^{jr}\int_0^t V^j(t)dt$
CF15	$\sum_{o=1}^{\lvert O\rvert}PSW'(t;E_o^r,\omega_o^r,t_o^r,1)$
CF16	$PSW(t;\varsigma^r\sum_{j=1}^{\lvert J\rvert}\sum_{m=1}^{\lvert M(j)\rvert}P_m^{jr}D_m^j,\hat{\omega}^r,\hat{t}_m^{jr},0)$
CF17	$\sum_{j=1}^{\lvert J\rvert}\sum_{i=1}^{\lvert I\rvert}PSW(t;\pi_i^{jr}g_i^jD_i,\omega_i,t_i',x_i')$

SOTF of sales tax are given parameters. Note that the sales tax startup time, $\hat{t}_m^{jr}$, is the first tax payment date after $t_m^j + \Delta t_m^j$ for $j \in J_s \subset J$ and $r \in R_s \subset R$, where J_s and R_s are the subsets of storage units and currencies, respectively, that a subsidiary $s \in S$ uses, where S is the set of subsidiaries of an MNC.

The average currency flow rate of customs duty of the material in storage unit j used by process i and imported to the nation that uses currency r is $\pi_i^{jr} g_i^j D_i$, where π_i^{jr}(currency/L) is the customs duty rate. Note that the customs duty payment includes the product yield of process i, g_i^j, because customs duty is paid to the destination nation of the goods where process i is a transportation process. Note that the variable operating cost—which is the process operating cost that is proportional to the average material flow rate through the process—can be treated in the same way as the customs duty.

Real business perform more sophisticated financial transactions, such as the pledging of account receivables, premature sales of marketable securities, and creditability accounting.[13] These factors are not considered in this study, but they could be by suitable modification of the PSW model.

Nonlinear Optimization Model

To obtain an analytical solution, it is necessary to assume that stockholder dividends and corporate income tax begin to be paid at the same time, that is,

$$t_o^r = t_{o'}^r (\equiv t_o^r) \qquad \forall o \neq o' \in O, r \in R \tag{9}$$

Invoking these assumptions simplifies the Lagrange multipliers in the solution using Kuhn-Tucker conditions. Table 2 lists the functional forms of CF1–CF17 obtained using Eqs. 1 and and 2 with the defined variables and parameters. Define $C^r(0)$ as the initial currency inventory and $C^r(t)$ as the currency inventory at time t. Then, the currency inventory at time t is calculated by adding the incoming flows (CF1–CF4) to the initial currency inventory and subtracting the outgoing flows (CF5–CF17):

$$\begin{aligned}
C^r(t) = {} & C^r(0) + \sum_{j=1}^{\lvert J\rvert}\sum_{m=1}^{\lvert M(j)\rvert}(1+\zeta^r)P_m^{jr}PSW(t;D_m^j,\omega_m^j,t_m^j+\Delta t_m^j,x_m^j) \\
& - \sum_{j=1}^{\lvert J\rvert}\sum_{k=1}^{\lvert K(j)\rvert}P_k^{jr}PSW(t;D_k^j,\omega_k^j,t_k^j+\Delta t_k^j,x_k^j) \\
& - \sum_{j=1}^{\lvert J\rvert}\sum_{k\in\{D_k^j\}^+}^{\lvert K(j)\rvert}PSW'(t;A_k^{jr},\omega_k^j,t_k^j,x_k^j) - \sum_{i\in\{D_i\}^+}^{\lvert I\rvert}PSW'(t;A_i^r,\omega_i,'t_i,'x_l) \\
& - \sum_{j=1}^{\lvert J\rvert}h^{jr}\int_0^t V^j(t)dt - \sum_{o=1}^{\lvert O\rvert}PSW'(t;E_o^r,\omega_o^r,t_o^r,1) \\
& + \sum_{n=1}^{\lvert N\rvert}PSW(t;E_n^r(1+\kappa_n^r\Delta t_n^r),\omega_n^r,t_n^r+\Delta t_n^r,0) \\
& - \sum_{n=1}^{\lvert N\rvert}PSW(t;E_n^r,\omega_n^r,t_n^r,0) - \sum_{n=1}^{\lvert N\rvert}PSW(t;E_n^r(1+\kappa_l^r\Delta t_l^r),\omega_l^r,t_l^r \\
& + \Delta t_l^r,0) - \sum_{l=1}^{\lvert L\rvert}PSW(t;E_l^r,\omega_l^r,t_l^r,0) - \sum_{n\in\{E_n^r\}^+}^{\lvert N\rvert}PSW'(t;A_n^r,\omega_n^r,t_n^r,0) \\
& - \sum_{l\in\{E_l^r\}^+}^{\lvert L\rvert}PSW'(t;A_l^r,\omega_l^r,t_l^r,0) - PSW(t;\zeta^r\sum_{j=1}^{\lvert J\rvert}\sum_{m=1}^{\lvert M(j)\rvert}P_m^{jr}D_m^j,\hat{\omega}^r,\hat{t}_m^{jr},0) \\
& + \sum_{r'\neq r}^{\lvert R\rvert}\chi^{r'r}PSW(t;E^{r'r},\omega^{r'r},t^{r'r},0) - \sum_{r'\neq r}^{\lvert R\rvert}PSW(t;E^{rr'},\omega^{rr'},t^{rr'},0) \\
& - \sum_{r,r'\in\{E^{r'r}\}^+}^{\lvert R\rvert}PSW'(t;A^{r'r},\omega^{r'r},t^{r'r},0) \\
& - \sum_{j=1}^{\lvert J\rvert}\sum_{i=1}^{\lvert I\rvert}PSW(t;\pi_i^{jr}g_i^jD_i,\omega_i,t_i',x_i') \qquad \forall r
\end{aligned} \tag{10}$$

where $\{D_k^j\}^+ \equiv \{k \mid D_k^j > 0\}$, $\{D_i\}^+ \equiv \{i \mid D_i > 0\}$, $\{E_n{}^r\}^+ \equiv \{n \mid E_n{}^r > 0\}$, $\{E_l{}^r\}^+ \equiv \{l \mid E_l{}^r > 0\}$, and $\{E^{r'r}\}^+ \equiv \{r \neq r' \mid E^{r'r} > 0\}$; that is, the index sets with positive average flow rates. Note that the SOTFs of pure financial transactions x_n^r, x_l^r, $x^{r'r}$, $x^{rr'}$, and $\hat{x}^r$ are set to zero without loss of generality. The SOTF of dividend to stockholders x_o^r is set to one for conformity with the continuous flow assumption. The average level of the currency inventory ($\overline{C}^r$) and the lower bound of the currency inventory ($\underline{\underline{C}}^r$) are easily calculated using the equations in Table 1:

$$\overline{C}^r = C^r(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\zeta^r)P_m^{jr}\overline{PSW}(t;D_m^j,\omega_m^j,t_m^j+\Delta t_m^j,x_m^j) - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}\overline{PSW}(t;D_k^j,\omega_k^j,t_k^j+\Delta t_k^j,x_k^j)$$
$$-\sum_{i\in\{D_i\}^+}\overline{PSW'}(t;A_i^r,\omega_i,{}'t_i,{}'x_I) - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\overline{PSW'}(t;A_k^{jr},\omega_k^{jr},t_k^j,x_k^j)$$
$$-\sum_{j=1}^{|J|}h^{jr}\overline{V^j}t - \sum_{n=1}^{|N|}\overline{PSW}(t;E_n^r,\omega_n^r,t_n^r,0) + \sum_{n=1}^{|N|}\overline{PSW}(t;E_n^r(1+\kappa_n^r\Delta t_n^r),\omega_n^r,t_n^r+\Delta t_n^r,0)$$
$$+\sum_{l=1}^{|L|}\overline{PSW}(t;E_l^r,\omega_l^r,t_l^r,0) - \sum_{l=1}^{|L|}\overline{PSW}(t;E_l^r(1+\kappa_l^r\Delta t_l^r),\omega_l^r,t_l^r+\Delta t_l^r,0)$$
$$-\sum_{n\in\{E_n^r\}^+}^{|N|}\overline{PSW'}(t;A_n^r,\omega_n^r,t_n^r,0) - \sum_{l\in\{E_l^r\}^+}^{|L|}\overline{PSW'}(t;A_l^r,\omega_l^r,t_l^r,0)$$
$$-\overline{PSW}(t;\zeta^r\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j,\hat{\omega}^r,\hat{t}_m^{jr},0) - \sum_{o=1}^{|O|}\overline{PSW'}(t;E_o^r,\omega_o^r,t_o^r,1)$$
$$+\sum_{r'\neq r}^{|R|}\chi^{r'r}\overline{PSW}(t;E^{r'r},\omega^{r'r},t^{r'r},0) - \sum_{r'\neq r}^{|R|}\overline{PSW}(t;E^{rr'},\omega^{rr'},t^{rr'},0)$$
$$-\sum_{r,r'\in\{E^{r'r}\}^+}^{|R|}\overline{PSW'}(t;A^{r'r},\omega^{r'r},t^{r'r},0) - \sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\overline{PSW}(t;\pi_i^{jr}\,g_i^jD_i,\omega_i,t_i',x_i') \quad \forall r \quad (11)$$

$$\underline{\underline{C}}^r = C^r(0)\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\varsigma^r)P_m^{jr}\underline{\underline{PSW}}(t;D_m^j,\omega_m^j,t_m^j+\Delta t_m^j,x_m^j) - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}\overline{\overline{PSW}}(t;D_k^j,\omega_k^j,t_k^j+\Delta t_k^j,x_k^j)$$
$$-\sum_{i\in\{D_i\}^+}\overline{\overline{PSW'}}(t;A_i^r,\omega_i,{}'t_i,{}'x_I) - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\overline{\overline{PSW'}}(t;A_k^{jr},\omega_k^j,t_k^j,x_k^j)$$
$$-\sum_{j=1}^{|J|}h^{jr}\overline{\overline{V^j}}t - \sum_{n=1}^{|N|}\overline{\overline{PSW}}(t;E_n^r,\omega_n^r,t_n^r,0) + \sum_{n=1}^{|N|}\underline{\underline{PSW}}(t;E_n^r(1+\kappa_n^r\Delta t_n^r),\omega_n^r,t_n^r+\Delta t_n^r,0)$$
$$+\sum_{l=1}^{|L|}\underline{\underline{PSW}}(t;E_l^r,\omega_l^r,t_l^r,0) - \sum_{l=1}^{|L|}\overline{\overline{PSW}}(t;E_l^r(1+\kappa_l^r\Delta t_l^r),\omega_l^r,t_l^r + \Delta t_l^r,0) - \sum_{n\in\{E_n^r\}^+}^{|N|}\overline{\overline{PSW'}}(t;A_n^r,\omega_n^r,t_n^r,0) - \sum_{l\in\{E_l^r\}^+}^{|L|}\overline{\overline{PSW'}}(t;A_l^r,\omega_l^r,t_l^r,0)$$
$$-\overline{\overline{PSW}}(t;\varsigma^r\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j,\hat{\omega}^r,\hat{t}_m^{jr},0) - \sum_{o=1}^{|O|}\overline{\overline{PSW'}}(t;E_o^r,\omega_o^r,t_o^r,1)$$
$$+\sum_{r'\neq r}^{|R|}\chi^{r'r}\underline{\underline{PSW}}(t;E^{r'r},\omega^{r'r},t^{r'r},0) - \sum_{r'\neq r}^{|R|}\overline{\overline{PSW}}(t;E^{rr'},\omega^{rr'},t^{rr'},0)$$
$$-\sum_{r,r'\in\{E^{r'r}\}^+}^{|R|}\overline{\overline{PSW'}}(t;A^{r'r},\omega^{r'r},t^{r'r},0) - \sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\overline{\overline{PSW}}(t;\pi_i^{jr}\,g_i^jD_i,\omega_i,t_i',x_i') \quad \forall r \quad (12)$$

We assume that the long-term currency in-flows and outflows for each currency storage unit are balanced. The average flow rates of currency into and out of a currency storage unit satisfy the following balance equation:

$$\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j + \sum_{n=1}^{|N|}\kappa_n^r\Delta t_n^rE_n^r + \sum_{r'\neq r}^{|R|}\chi^{r'r}E^{r'r} = \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}D_k^j + \sum_{i\in\{D_i\}^+}^{|I|}\frac{A_i^r}{\omega_i} + \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\frac{A_k^{jr}}{\omega_k^j} + \sum_{o=1}^{|O|}E_o^r$$
$$+\sum_{l=1}^{|L|}\kappa_l^r\Delta t_l^rE_l^r + \sum_{l\in\{E_l\}^+}^{|L|}\frac{A_l^r}{\omega_l^r} + \sum_{n\in\{E_n\}^+}^{|N|}\frac{A_n^r}{\omega_n^r} + \sum_{j=1}^{|J|}h^{jr}\overline{V^j} + \sum_{r'\neq r}^{|R|}E^{rr'} + \sum_{r,r'\in\{E^{r'r}\}^+}^{|R|}\frac{A^{r'r}}{\omega^{r'r}} + \sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\pi_i^{jr}\,g_i^jD_i \qquad \forall r \quad (13)$$

Corporate income tax is computed based on the gross profit with depreciation charges:

$$\frac{1}{\xi^r}(E_{\bar{o}}^r) = \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}D_k^j + \sum_{j=1}^{|J|}\sum_{i\notin I_r}^{|I|}P_i^{jr}(f_i^j - g_i^j)D_i - \sum_{r'\neq r}^{|R|}\sum_{j=1}^{|J|}\sum_{i\in I_r}^{|I|}\chi^{r'r}P_i^{jr'}(f_i^j-g_i^j)D_i$$
$$+\sum_{n=1}^{|N|}\kappa_n^r\Delta t_n^rE_n^r - \sum_{l=1}^{|L|}\kappa_l^r\Delta t_l^rE_l^r - \sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\pi_i^{jr}\,g_i^jD_i - \sum_{j=1}^{|J|}h^{jr}\overline{V^j} - \sum_{i\in\{D_i\}^+}^{|I|}\frac{A_i^r}{\omega_i} - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\frac{A_k^{jr}}{\omega_k^j} - \sum_{l\in\{E_l\}^+}^{|L|}\frac{A_l^r}{\omega_l^r}$$
$$-\sum_{n\in\{E_n\}^+}^{|N|}\frac{A_n^r}{\omega_n^r} - \sum_{r,r'\in\{E^{r'r}\}^+}^{|R|}\frac{A^{r'r}}{\omega^{r'r}} - \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}a_k^{jr}D_k^j\omega_k^j - \sum_{i=1}^{|I|}a_i^rD_i\omega_i - \sum_{j=1}^{|J|}b^{jr}\overline{\overline{V^j}} \quad (14)$$

where $E_{\bar{o}}^r$ is the average flow rate of corporate income tax at a rate of ξ^r paid in currency r. A nonprofitable subsidiary would not pay corporate income tax, and hence $\xi^r = 0$ for $r \in R_s$ if $\sum_{r\in R_s}E_{\bar{o}}^r \leq 0$. The right-hand side of Eq. 14 include two fictitious terms $\sum_{j=1}^{|J|}\sum_{i\notin I_r}^{|I|}P_i^{jr}(f_i^j - g_i^j)D_i$ and $-\sum_{r'\neq r}^{|R|}$

$\sum_{j=1}^{|J|}\sum_{i\in I_r}^{|I|}\chi^{r'r}\ P_i^{jr'}(f_i^j-g_i^j)D_i$, where P_i^{jr} (currency/L) is the transfer price of the material in storage j via process i paid in currency r, and $I_r\subset I$ is the process subset owned by the nation that uses currency r. P_i^{jr} is set to zero if process i is not related to the storage units at different nations. Note that $j\in J_r$ in $P_i^{jr},P_k^{jr},P_m^{jr},A_k^{jr},\pi_i^{jr}$, and γ^{jr} where $J_r\subset J$ is the storage subset owned by the nation that uses currency r. The transfer price is the price that a selling subsidiary of an MNC charges for a product supplied to a buying subsidiary of the same MNC. The transfer price should be chosen to minimize the overall income tax of an MNC under the constraints of official regulations.[14] A subsidiary of an MNC in a nation pays corporate income tax according to that nation's tax regulations. Therefore, the values of all the materials moving into or out of the subsidiary need to be properly accounted. Actual currency transfer may be accompanied by the material movement between subsidiaries based on transfer price, which yields $\chi^{r'r}E^{r'r}\geq\sum_{j=1}^{|J|}\sum_{i\in I_{r'}}^{|I|}P_i^{jr}f_i^jD_i+\sum_{j=1}^{|J|}\times\sum_{i\in I_r}^{|I|}\chi^{r'r}\ P_i^{jr'}g_i^jD_i$. The terms $\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}a_k^{jr}D_k^j\omega_k^j$, $\sum_{i=1}^{|I|}\times a_i^rD_i\omega_i$, and $\sum_{j=1}^{|J|}b^{jr}\overline{\overline{V^j}}$ are the annualized depreciations for processes and storage units. The annualized capital investment cost and annualized depreciation of facilities are matched for conformity with assumption (iv). If this is not the case, new variables for depreciations should be introduced and the final results of this study (Eqs. 25 and 26) will be slightly different.

Then, by using the equations in Table 1 and Eq. 11, 12, and 13 can be simplified to the following forms:

$$\begin{aligned}\overline{C^r}=&\,C^r(0)+\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\varsigma^r)P_m^{jr}[0.5(1-x_m^j)D_m^j\omega_m^j\\&-D_m^j(t_m^j+\Delta t_m^j)]-\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}[0.5(1-x_k^j)D_k^j\omega_k^j\\&-D_k^j(t_k^j+\Delta t_k^j)]-\sum_{i\in\{D_i\}^+}^{|I|}A_i^r\left[0.5(1-{}'x_i)-\frac{{}'t_i}{\omega_i}\right]\\&-\sum_{j=1}^{|J|}\sum_{k\in\{D_i^j\}^+}^{|K(j)|}A_k^{jr}\left[0.5(1-x_k^j)-\frac{t_k^j}{\omega_k^j}\right]-\sum_{n\in\{E_n\}^+}^{|N|}A_n^r\left[0.5-\frac{t_n^r}{\omega_n^r}\right]\\&-\sum_{l\in\{E_l\}^+}^{|L|}A_l^r\left[0.5-\frac{t_l^r}{\omega_l^r}\right]-\sum_{n=1}^{|N|}\Big[\kappa_n^r\Delta t_n^rt_n^r+\kappa_n^r(\Delta t_n^r)^2+\Delta t_n^r\\&-0.5\kappa_n^r\Delta t_n^r\omega_n^r\Big]E_n^r+\sum_{l=1}^{|L|}\Big[K_l^r\Delta t_l^rt_l^r+\kappa_l^r(\Delta t_l^r)^2+\Delta t_l^r\\&-0.5\kappa_l^r\Delta t_l^r\omega_l^r\Big]E_l^r-\varsigma^r\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j[0.5\hat{\omega}^r-\hat{t}_m^{jr}]\\&+\sum_{o=1}^{|O|}t_o^rE_o^r+\sum_{r'\neq r}^{|R|}\chi^{r'r}[0.5E^{r'r}\omega^{r'r}-E^{r'r}t^{r'r}]-\sum_{r'\neq r}^{|R|}[0.5E^{rr'}\omega^{rr'}\\&-E^{rr'}t^{rr'}]-\sum_{r,r'\in\{E^{r'r}\}^+}^{|R|}A^{r'r}\left[0.5-\frac{t^{r'r}}{\omega^{r'r}}\right]\\&-\sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\pi_i^{jr}g_i^jD_i[0.5(1-x_i')\omega_i-t_i']\qquad\forall r\qquad(15)\end{aligned}$$

$$\begin{aligned}\underline{\underline{C^r}}=&\,C^r(0)-\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\varsigma^r)P_m^{jr}[D_m^j(t_m^j+\Delta t_m^j)]\\&-\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}[(1-x_k^j)D_k^j\omega_k^j-D_k^j(t_k^j+\Delta t_k^j)]\\&-\sum_{i\in\{D_i\}^+}^{|I|}A_i^r\left[(1-{}'x_i)-\frac{{}'t_i}{\omega_i}\right]-\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}A_k^{jr}\left[(1-x_k^j)-\frac{t_k^j}{\omega_k^j}\right]\\&-\sum_{j=1}^{|J|}h^{jr}\left[\overline{\overline{V^j}}-\overline{V^j}\right]-\sum_{n=1}^{|N|}\Big[\kappa_n^r\Delta t_n^rt_n^r+\kappa_n^r(\Delta t_n^r)^2+\Delta t_n^r+\omega_n^r\Big]E_n^r\\&-\sum_{n\in\{E_n\}^+}^{|N|}A_n^r\left(1-\frac{t_n^r}{\omega_n^r}\right)+\sum_{l=1}^{|L|}\Big[\kappa_l^r\Delta t_l^rt_l^r+\kappa_l^r(\Delta t_l^r)^2+\Delta t_l^r\\&-(1+\kappa_l^r\Delta t_l^r)\omega_l^r\Big]E_l^r-\sum_{l\in\{E_l\}^+}^{|L|}A_l^r\left(1-\frac{t_l^r}{\omega_l^r}\right)+\sum_{o=1}^{|O|}t_o^rE_o^r\\&-\varsigma^r\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j[\hat{\omega}^r-\hat{t}_m^{jr}]-\sum_{r'\neq r}^{|R|}\chi^{r'r}E^{r'r}t^{r'r}\\&-\sum_{r'\neq r}^{|R|}[E^{rr'}\omega^{rr'}-E^{rr'}t^{rr'}]-\sum_{r,r'\in\{E^{r'r}\}^+}^{|R|}A^{r'r}\left(1-\frac{t^{r'r}}{\omega^{r'r}}\right)\\&-\sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\pi_i^{jr}\ g_i^jD_i[(1-x_i')\omega_i-t_i']\qquad\forall r\qquad(16)\end{aligned}$$

Note that $t_o^r=t_{\bar{o}}^r$ and the term $\sum_{o=1}^{|O|}E_o^r$ in Eqs. 15 and 16 can be further developed using Eq. 13. Equation 15 is used to compute the opportunity cost of the currency inventory, and Eq. 16 should be nonnegative to ensure that the currency storage is not depleted, because a shortage of currency will incur severe additional costs or even bankruptcy. Therefore, $\underline{\underline{V^j}}\geq0$ and $\underline{\underline{C^r}}\geq0$ constitute the constraints of the design optimization.

Suppose η^r(currency/currency/year) is the rate of the opportunity cost of the currency inventory. The objective function of the optimization is to minimize the annualized opportunity costs of capital investment for processing/storage units and currency/material inventories minus the dividend to stockholders expressed in the numeraire currency ($r=1$):

$$\begin{aligned}Minimize\ TC=&\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\chi^{r1}a_k^{jr}D_k^j\omega_k^j+\sum_{r=1}^{|R|}\sum_{i=1}^{|I|}\chi^{r1}a_i^rD_i\omega_i\\&+\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}b^{jr}\overline{\overline{V^j}}+\sum_{r=1}^{|R|}\chi^{r1}\eta^r\overline{C}^r+\sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\gamma^{jr}\overline{V^j}\\&-\sum_{r=1}^{|R|}\sum_{o\neq\bar{o}}^{|O|}\chi^{r1}E_o^r\qquad(17)\end{aligned}$$

where

$$\begin{aligned}\sum_{o\neq\bar{o}}^{|O|}E_o^r=&\,(1-\xi^r)\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j+(1-\xi^r)\sum_{n=1}^{|N|}\kappa_n^r\Delta t_n^rE_n^r\\&+\sum_{r'\neq r}^{|R|}\chi^{r'r}E^{r'r}-(1-\xi^r)\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}D_k^j-(1-\xi^r)\sum_{i\in\{D_i\}^+}^{|I|}\frac{A_i^r}{\omega_i}\end{aligned}$$

$$- (1-\xi^r)\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\frac{A_k^{jr}}{\omega_k^j} - (1-\xi^r)\sum_{l=1}^{|L|}\kappa_l^r\Delta t_l^r E_l^r - (1-\xi^r)$$

$$\times \sum_{l\in\{E_l\}^+}^{|L|}\frac{A_l^r}{\omega_l^r} - (1-\xi^r)\sum_{n\in\{E_n\}^+}^{|N|}\frac{A_n^r}{\omega_n^r} - (1-\xi^r)\sum_{j=1}^{|J|}h^{jr}\overline{V^j}$$

$$- \sum_{r'\neq r}^{|R|}E^{rr'} - (1-\xi^r)\sum_{r'\neq r}^{|R|}\frac{A^{r'r}}{\omega^{r'r}} - (1-\xi^r)\sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\pi_i^{jr}g_i^jD_i$$

$$+ \xi^r\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}a_k^{jr}D_k^j\omega_k^j + \xi^r\sum_{i=1}^{|I|}a_i^rD_i\omega_i + \xi^r\sum_{j=1}^{|J|}b^{jr}\overline{\overline{V^j}}$$

$$- \xi^r\sum_{j=1}^{|J|}\sum_{i\notin I_r}^{|I|}P_i^{jr}(f_i^j - g_i^j)D_i + \xi^r\sum_{r'\neq r}^{|R|}\sum_{j=1}^{|J|}\sum_{i\in I_r}^{|I|}\chi^{r'r}P_i^{jr'}(f_i^j - g_i^j)D_i \quad \forall r \qquad (18)$$

By using Eq. 18, Eq. 17 can be rewritten as

$$TC = \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\chi^{r1}\left[(1-\xi^r)\frac{A_k^{jr}}{\omega_k^j} + (1-\xi^r)a_k^{jr}D_k^j\omega_k^j + (1-\xi^r)P_k^{jr}D_k^j\right] + \sum_{r=1}^{|R|}\sum_{i=1}^{|I|}\chi^{r1}\left[(1-\xi^r)\frac{A_i^r}{\omega_i} + (1-\xi^r)a_i^rD_i\omega_i + (1-\xi^r)\sum_{j=1}^{|J|}\pi_i^{jr}g_i^jD_i\right] + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\xi^r\left[\sum_{i\notin I_r}^{|I|}P_i^{jr}\left(f_i^j - g_i^j\right)D_i - \sum_{r'\neq r}^{|R|}\sum_{i\in I_r}^{|I|}\chi^{r'r}P_i^{jr'}\left(f_i^j - g_i^j\right)D_i\right] + \sum_{r=1}^{|R|}\sum_{n=1}^{|N|}\chi^{r1}(1-\xi^r)\frac{A_n^r}{\omega_n^r}$$

$$+ \sum_{r=1}^{|R|}\sum_{l=1}^{|L|}\chi^{r1}(1-\xi^r)\frac{A_l^r}{\omega_l^r} + \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\chi^{r'1}(1-\xi^{r'})\frac{A^{rr'}}{\omega^{rr'}} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\left[\left((1-\xi^r)h^{jr} + \gamma^{jr}\right)\overline{V^j} + (1-\xi^r)b^{jr}\overline{\overline{V^j}}\right]$$

$$+ \sum_{r=1}^{|R|}\sum_{l=1}^{|L|}\chi^{r1}(1-\xi^r)\kappa_l^r\Delta t_l^rE_l^r - \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\chi^{r1}(1-\xi^r)P_m^{jr}D_m^j + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\overline{C}^r + \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\chi^{r1}E^{rr'} - \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\chi^{r'1}\chi^{rr'}E^{rr'} - \sum_{r=1}^{|R|}\sum_{n=1}^{|N|}\chi^{r1}(1-\xi^r)\kappa_n^r\Delta t_n^rE_n^r \qquad (19)$$

Without loss of generality, the material storage size is determined by the upper bound of the inventory holding $\overline{\overline{V^j}}$. Therefore, Eq. 6 is the expression for the storage capacity. The independent variables are selected to be the cycle times (ω_k^j, ω_i, ω_n^r, ω_l^r, and $\omega^{rr'}$), start-up times ($t_{\bar{o}}^r$, t_k^j, $'t_i$, t_n^r, t_l^r, and $t^{rr'}$), and average material/currency flow rates (D_k^j, D_i, E_n^r, E_l^r, and $E^{rr'}$). Note that the startup time t_i' is converted into $'t_i$ by Eq. 3.

The objective function in Eq. 19 is convex, and the constraints are linear with respect to ω_k^j, ω_i, ω_n^r, ω_l^r, $\omega^{rr'}$, $t_{\bar{o}}^r$, t_k^j, $'t_i$, t_n^r, t_l^r, and $t^{rr'}$ if D_k^j, D_i, E_n^r, E_l^r, and $E^{rr'}$ are considered as parameters. However, the convexity with respect to D_k^j, D_i, E_n^r, E_l^r, and $E^{rr'}$ is not clear. We first obtain the solution for the Kuhn-Tucker conditions with respect to ω_k^j, ω_i, ω_n^r, ω_l^r, $\omega^{rr'}$, $t_{\bar{o}}^r$, t_k^j, $'t_i$, t_l^r, and $t^{rr'}$ when D_k^j, D_i, E_n^r, E_l^r, and $E^{rr'}$ are considered as parameters, and then we further solve the problem with respect to D_k^j, D_i, E_n^r, E_l^r, and $E^{rr'}$. Although the problem is thereby separated into a two-level parametric optimization problem, the Kuhn-Tucker conditions of the original problem and the two-level problem are the same if the constraints are reduced to equalities[1]. In other words, the Kuhn-Tucker conditions of the first-level problem produce an explicit analytical solution and the original problem can be reduced to the second-level problem by eliminating the design variables of the first-level problem. The first-level problem of the two-level problem has a convex objective function with linear inequality constraints, and the second-level problem has a nonconvex objective function with nonlinear equality constraints. The two-level parametric approach yields a global optimum inasmuch as the second-level problem converges to its global optimum.

Solution of Kuhn-Tucker Conditions

The solution of the Kuhn-Tucker conditions of the first-level optimization problem, which entails minimizing the objective function in Eq. 19 subject to the constraints $\underline{\underline{V^j}} \geq 0$ and $\underline{\underline{C}}^r \geq 0$ with fixed values of D_k^j, D_i, E_n^r, E_l^r, and $\overline{E^{rr'}}$ is obtained by the algebraic manipulation summarized in Appendix A. The optimal cycle times are

$$\omega_k^j = \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)}{D_k^j\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^{jr}\right)}} \quad \forall j,k \qquad (20)$$

$$\omega_i = \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)}{D_i\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)}} \quad \forall i \qquad (21)$$

$$\omega_n^r = \sqrt{\frac{(1-\xi^r)A_n^r}{E_n^r\Psi_n^r}} \quad \forall n,r \qquad (22)$$

$$\omega_l^r = \sqrt{\frac{(1-\xi^r)A_l^r}{E_l^r\Psi_l^r}} \quad \forall l,r \qquad (23)$$

$$\omega^{rr'} = \sqrt{\frac{(1-\xi^{r'})A^{rr'}}{E^{rr'}\Psi^{rr'}}} \quad \forall r,r' \qquad (24)$$

where

$$\Psi_k^{jr} = \left[\left(\frac{(1-\xi^r+\eta^r)h^{jr} + \gamma^{jr} + \eta^rP_k^{jr}}{2} + (1-\xi^r)b^{jr}\right) \times (1-x_k^j) + (1-\xi^r)a_k^{jr}\right] \quad \forall j,k,r \qquad (25)$$

$$\Psi_i^r = \Big[(1-\xi^r)a_i^r + (1-{}'x_i)$$
$$\times \sum_{j=1}^{|J|}\left(\frac{(1-\xi^r+\eta^r)h^{jr}+\gamma^{jr}}{2}+(1-\xi^r)b^{jr}\right)f_i^j\Big] + \Big[(1-x_i')$$
$$\times \sum_{j=1}^{|J|}\left(\frac{(1-\xi^r+\eta^r)h^{jr}+\gamma^{jr}+\eta^r\pi_i^{jr}}{2}+(1-\xi^r)b^{jr}\right)g_i^j\Big] \quad \forall i,r \quad (26)$$

$$\Psi_n^r = \eta^r(1+0.5\kappa_n^r\Delta t_n^r) \quad \forall n,r \quad (27)$$

$$\Psi_l^r = \eta^r(1+0.5\kappa_l^r\Delta t_l^r) \quad \forall l,r \quad (28)$$

$$\Psi^{rr'} = 0.5\left(\eta^{r'}\chi^{rr'}+\eta^r\left(\frac{\chi^{r1}}{\chi^{r'1}}\right)\right) \quad \forall r,r' \quad (29)$$

Because the values of the multipliers are positive, Eq. A12 gives $\underline{\underline{V^j}} = 0$ and $\underline{\underline{C^r}} = 0$. Equations 7 and 16 yield the following expressions:

$$\sum_{k=1}^{|K(j)|}D_k^j t_k^j + \sum_{i=1}^{|I|}(g_i^j - f_i^j)D_i{}'t_i = V^j(0) - \sum_{m=1}^{|M(j)|}(1-x_m^j)D_m^j\omega_m^j$$
$$+\sum_{m=1}^{|M(j)|}D_m^j t_m^j - \sum_{i=1}^{|I|}g_i^j D_i\Delta t_i - \sum_{i=1}^{|I|}(1-{}'x_i)f_i^j$$
$$\times\sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)D_i}{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)}} \quad \forall j \quad (30)$$

$$\sum_{n=1}^{|N|}\left\{\{t_{\bar{o}}^r - t_l^r\} + \frac{(1-\xi^r)}{\Psi_n^r}\right\}\sqrt{\frac{A_n^r\Psi_n^r E_n^r}{(1-\xi^r)}} + \sum_{n=1}^{|N|}\big[\Delta t_n^r(1+\kappa_n^r\Delta t_n^r)$$
$$-\kappa_n^r\Delta t_n^r\{t_{\bar{o}}^r - t_n^r\}\big]E_n^r + \sum_{i=1}^{|L|}\left\{\{t_{\bar{o}}^r - t_l^r\} + \frac{(1+\kappa_l^r\Delta t_l^r)(1-\xi^r)}{\Psi_l^r}\right\}$$
$$\times\sqrt{\frac{A_l^r\Psi_l^r E_l^r}{(1-\xi^r)}} - \sum_{l=1}^{|L|}\big[\Delta t_l^r(1+\kappa_l^r\Delta t_l^r) - \kappa_l^r\Delta t_l^r\{t_{\bar{o}}^r - t_l^r\}\big]E_l^r$$
$$+\sum_{r'\neq r}^{|R|}\left[\left(\chi^{r'r}E^{r'r} - \sqrt{\frac{A^{r'r}\Psi^{r'r}E^{r'r}}{(1-\xi^r)}}\right)\{t^{r'r} - t_{\bar{o}}^r\}\right]$$
$$+\sum_{r'\neq r}^{|R|}\left[\frac{\sqrt{(1-\xi^{r'})A^{rr'}\Psi^{rr'}E^{rr'}}}{\Psi^{rr'}} - E^{rr'}\{t^{rr'} - t_{\bar{o}}^r\}\right]$$
$$= C^r(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}P_m^{jr}D_m^j\big[t_{\bar{o}}^r - (1+\varsigma^r)(t_m^j + \Delta t_m^j) - \varsigma^r\{\hat{\omega}^r - \hat{t}_m^{jr}\}\big]$$
$$-\sum_{i\in\{D_i\}^+}^{|I|}\left[A_i^r(1-{}'x_i) + A_i^r\sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)D_i}{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)}}\{t_{\bar{o}}^r - {}'t_i\}\right]$$
$$-\sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\pi_i^{jr}g_i^j D_i\left[(1-x_i')\sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)}{D_i\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)}} - {}'t_i - \Delta t_i\right]$$
$$-\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^{jr}D_k^j\left\{(1-x_k^j)\sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^{jr}\right)}{D_k^j\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)}} + (t_{\bar{o}}^r - t_k^j - \Delta t_k^j)\right\}$$
$$-\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}A_k^{jr}\sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)D_k^j}{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)}}\{t_{\bar{o}}^r - t_k^j\}$$
$$-\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}A_k^{jr}(1-x_k^j) - \sum_{j=1}^{|J|}(1+t_{\bar{o}}^r)h^{jr}\left({}^*\overline{V^j}\right) - \sum_{n\in\{E_n^r\}^+}^{|N|}A_n^r$$
$$-\sum_{l\in\{E_l^r\}^+}^{|L|}A_l^r - \sum_{r'\neq r\in\{E^{r'r}\}^+}^{|R|}A^{r'r} \quad \forall r \quad (31)$$

where

$${}^*\overline{V^j} = \sum_{k=1}^{|K(j)|}\frac{(1-x_k^j)}{2}D_k^j\omega_k^j + \sum_{m=1}^{|M(j)|}\frac{(1-x_m^j)}{2}D_m^j\omega_m^j$$
$$+\sum_{i=1}^{|I|}\frac{(1-x_i')}{2}g_i^j D_i\omega_i + \sum_{i=1}^{|I|}\frac{(1-{}'x_i)}{2}f_i^j D_i\omega_i \quad \forall j \quad (32)$$

Equation 32 is derived from Eqs. 8 and 30. Equations 6 and 30 indicate that the optimal material storage size is ${}^*\overline{\overline{V^j}} = 2{}^*\overline{V^j}$. The optimal average level of currency storage, calculated using Eqs. 15 and 31, is

$${}^*\overline{C}^r = \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}0.5P_m^{jr}D_m^j\big[(1+\varsigma^r)(1-x_m^j)\omega_m^j + \varsigma^r\hat{\omega}^r\big]$$
$$+\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}0.5P_k^{jr}D_k^j(1-x_k^j)\omega_k^j + \sum_{i\in\{D_i\}^+}^{|I|}0.5A_i^r(1-{}'x_i)$$
$$+\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}0.5A_k^{jr}(1-x_k^j) + \sum_{j=1}^{|J|}h^{jr}\left({}^*\overline{V}^j\right)$$
$$+\sum_{n=1}^{|N|}(1+0.5\kappa_n^r\Delta t_n^r)\omega_n^r E_n^r + \sum_{l=1}^{|L|}(1+0.5\kappa_l^r\Delta t_l^r)\omega_l^r E_l^r$$
$$+\sum_{n\in\{E_n^r\}^+}^{|N|}0.5A_n^r + \sum_{l\in\{E_l^r\}^+}^{|L|}0.5A_l^r + 0.5\sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\pi_i^{jr}g_i^j$$
$$\times D_i(1-x_i')\omega_i + \sum_{r'\neq r}^{|R|}0.5\chi^{r'r}\omega^{r'r}E^{r'r} + \sum_{r'\neq r}^{|R|}0.5\omega^{rr'}E^{rr'}$$
$$+\sum_{r'\neq r\in\{E^{r'r}\}^+}^{|R|}0.5A^{r'r} \quad \forall r \quad (33)$$

Thus, the optimal currency storage size is ${}^*\overline{\overline{C}} = 2{}^*\overline{C}$. Then, the optimal value of the objective function (calculated from Eqs. 19 to 33) is

$$
\begin{aligned}
&{}^*TC(D_k^j, D_i, E_n^r, E_l^r, E^{rr'}) \\
&= 2\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)D_k^j} \\
&\quad + 2\sum_{i=1}^{|I|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)D_i} \\
&\quad + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\chi^{r1}(1-\xi^r)\pi_i^{jr}g_i^jD_i + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\xi^r \\
&\quad \times\left[\sum_{i\notin I_r}^{|I|}P_i^{jr}(f_i^j - g_i^j)D_i - \sum_{r'\neq r}^{|R|}\sum_{i\in r}^{|I|}\chi^{r'r}P_i^{jr'}(f_i^j - g_i^j)D_i\right] \\
&\qquad + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\chi^{r1}(1-\xi^r)P_k^{jr}D_k^j \\
&\quad + \sum_{r=1}^{|R|}\sum_{n=1}^{|N|}\chi^{r1}\left[2\sqrt{(1-\xi^r)A_n^r\Psi_n^rE_n^r} - \kappa_n^r\Delta t_n^r(1-\xi^r)E_n^r\right] \\
&\quad + \sum_{r=1}^{|R|}\sum_{l=1}^{|L|}\chi^{r1}\left[2\sqrt{(1-\xi^r)A_l^r\Psi_l^rE_l^r} + \kappa_l^r\Delta t_l^r(1-\xi^r)E_l^r\right] \\
&+ \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\chi^{r1}\left[\left(\frac{1-\xi^r)h^{jr} + \gamma^{jr} + \eta^r h^{jr} + \eta^r(1-\varsigma^r)P_m^{jr}}{2}\right.\right. \\
&\qquad \left.\left. + (1-\xi^r)b^{jr}\right)(1-x_m^j)\omega_m^j\right]D_m^j \\
&\quad + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\chi^{r1}P_m^{jr}D_m^j\left(0.5\eta^r\varsigma^r\hat{\omega}^r - (1-\xi^r)\right) \\
&\quad + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{l\in\{E_l^r\}^+}^{|L|}0.5A_l^r + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{r'\neq r\in\{E^{r'r}\}^+}^{|R|}0.5A^{r'r} \\
&\qquad + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{i\in\{D_i\}^+}^{|I|}0.5(1-{}'x_i)A_i^r + \sum_{r=1}^{|R|}\chi^{r1}\eta^r \\
&\quad \times\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}0.5(1-x_k^j)A_k^{jr} + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{n\in\{E_n^r\}^+}^{|N|}0.5A_n^r \\
&\quad + \sum_{r=1}^{|R|}\sum_{r'\neq r}^{|R|}\left[(\chi^{r1} - \chi^{r'1}\chi^{rr'})E^{rr'} + 2\chi^{r'1}\sqrt{(1-\xi^{r'})A^{rr'}\Psi^{rr'}E^{rr'}}\right]
\end{aligned}
\tag{34}
$$

The second-level optimization problem entails minimizing the objective function in Eq. 34 under the constraints in Eqs. 4, 30, and 31 with respect to the design variables D_k^j, D_i, E_n^r, E_l^r, $E^{rr'}$, $t_{\bar{o}}^r$, t_k^j, ${}'t_i$, t_n^r, t_l^r, and $t^{rr'}$. The second-level optimization problem is a nonconvex nonlinear programming formulation with bilinear terms, such as $D_k^jt_k^j$ and $D_i{}'t_i$ in Eq. 31, as well as concave terms (square roots). Some of the average flow rates are zero at the optimum condition, which makes it difficult to compute the derivatives of their square roots. Moreover, A_i^r, A_k^{jr}, A_n^r, A_l^r, and $A^{rr'}$ should be zero if their corresponding average flow rates go to zero at the optimum condition. To address this issue, the objective function in Eq. 34 should include binary variables to exclude the setup costs whose average flow rates become zero. An easy suboptimal procedure has been suggested for solving the second-level problem,[1] and is summarized as follows: The objective function in Eq. 34 is separable with respect to the material flow terms composed of D_k^j and D_i and the currency flow terms composed of E_n^r, E_l^r, and $E^{rr'}$. Minimizing the material flow terms in Eq. 34 with respect to D_k^j and D_i under the constraint of Eq. 4 gives average material flow rates of D_k^j and D_i. Then, cycle times ω_k^j and ω_i are computed using Eqs. 20 and 21. The startup times t_k^j and ${}'t_i$ are computed using Eq. 30, and $t_{\bar{o}}^r$ is computed using the equality in Eq. 37, as derived below. Finally, the average financial flows E_n^r, E_l^r, and $E^{rr'}$ are computed by minimizing the currency terms in Eq. 34 with respect to E_n^r, E_l^r, and $E^{rr'}$ under the constraint in Eq. 31. The terms in Eq. 31 become separable concave functions if t_n^r, t_l^r, and $t^{rr'}$ are already known. Many efficient algorithms already exist to solve optimization with separable concave functions.[1] The second-level optimization problem can be replaced with another model (e.g., an ordinary linear programming formulation) to compute the average rates of material and currency flows without impacting the optimality of the lot sizing equations derived from the first-level optimization problem.

Additional Constraints

Note that the value of $2\sqrt{(1-\xi^r)A_n^r\Psi_n^rE_n^r} - \kappa_n^r\Delta t_n^r(1-\xi^r)E_n^r$ in Eq. 34 should be nonpositive; otherwise E_n^r is zero at the optimum point:

$$E_n^r = 0 \text{ or } \frac{4A_n^r\Psi_n^r}{(1-\xi^r)(\kappa_n^r\Delta t_n^r)^2} \le E_n^r \quad \forall n, r \tag{35}$$

There is a self-motivation to transfer currency if there is a financial arbitrage larger than the transfer cost such that the value of currency converted in the sequence $r \to r' \to 1$ is better than the value of currency converted in the sequence $r \to 1$:

$$\chi^{r1} < X^{r'1}\chi^{rr'} \text{ and } \frac{4(1-\xi^{r'})A^{rr'}\Psi^{rr'}}{\left(\frac{\chi^{r1}}{\chi^{r'1}} - \chi^{rr'}\right)^2} \le E^{rr'} \quad \forall r.r' \tag{36}$$

At the initial stage of a profitable manufacturing business, the condition $\underline{\underline{C}}^r \ge 0$ should be satisfied even in the absence of temporary financial investments, bank loans, and currency transfers. For $E_n^r = E_l^r = E^{rr'} = 0$, Eq. 16 gives the following inequality with respect to $t_{\bar{o}}$:

$$t_{\bar{o}}^r \ge \frac{(\text{Numerator})}{(\text{Denominator})} \quad \forall r \tag{37}$$

where

$$\text{Numerator} = -C^r(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr} D_m^j [(1+\zeta^r)(t_m^j + \Delta t_m^j)$$

$$+ \zeta^r\{\hat{\omega}^r - \hat{t}_m^{jr}\}] + \sum_{i\in\{D_i\}^+}^{|I|} \left[A_i^r(1-'x_l) - 't_i A_i^r \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)D_i}{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)}} \right]$$

$$+ \sum_{i=1}^{|I|}\sum_{j=1}^{|J|} \pi_i^{jr} g_i^j D_i \left[(1-x_i') \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)}{D_i\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)}} - 't_i - \Delta t_i \right]$$

$$+ \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^{jr} D_k^j \left\{ (1-x_k^j) \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)}{D_k^j\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)}} - (t_k^j + \Delta t_k^j) \right\}$$

$$+ \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|} A_k^{jr} \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)D_k^j}{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)}} t_k^j$$

$$- \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|} A_k^{jr}(1-x_k^j) - \sum_{j=1}^{|J|} h^{jr}({}^*\overline{V^J}) \quad \forall r$$

$$\text{Denominator} = \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} P_m^{jr} D_m^j$$

$$- \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} \left[A_k^{jr} \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_k^{jr}\right)D_k^j}{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}\right)}} + P_k^{jr} D_k^j \right]$$

$$- \sum_{i=1}^{|I|} \left[A_i^r \sqrt{\frac{\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)D_i}{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)}} + \sum_{j=1}^{|J|} \pi_i^{jr} g_i^j D_i \right]$$

$$- \sum_{j=1}^{|J|} h^{jr}({}^*\overline{V^j}) \qquad \forall r$$

The equality in Eq. 37 gives the optimal value of $t_{\bar{o}}$ for a given $C^r(0)$ and $E_n^r = E_l^r = E^{rr'} = 0$. Eq. 37 is an optional one that can have different forms depending on circumstances. Unfortunately, neither Eq. 31 nor 37, guarantees the absence of an initial shortage of currency inventory, which is mostly caused by paying for raw materials before receiving income from product sales. The initial currency inventory $C^r(0)$ should cover such a currency shortage. Let us define $t_c^r = \min_{j\in J_s}\{t_m^j + \Delta t_m^j\}$. Then,

$$C^r(0) \geq \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|} P_k^{jr} PSW(t_c^r; D_k^j, \omega_k^j, t_k^j + \Delta t_k^j, x_k^j)$$

$$+ \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|} PSW'(t_c^r; A_k^{jr}, \omega_k^j, t_k^j, x_k^j)$$

$$+ \sum_{i\in\{D_i\}^+}^{|I|} PSW'(t_c^r; A_i^r, \omega_i, 't_i, 'x_l) + \sum_{j=1}^{|J|} h^{jr} \int_0^{t_c} V^j(t)dt \quad \forall r \qquad (38)$$

Minimizing the initial currency inventory is one of the hidden design criteria. $C^r(0)$ can be determined from the equality in Eq. 38, which may include additional terms from Eq. 10 depending on the circumstances. If $C^r(0)$ is a given parameter whose value is insufficient to cover the initial currency requirement, Eq. 38 acts as a severe constraint with respect to the design variables.

If the cycle times and/or startup times are outside the upper or lower bound, the bound that is closest to the unbounded optimal solution should be selected because the objective function is convex with respect to the cycle times and the startup times in the first-level optimization problem. In addition, the upper and lower bounds of the average flow rates can be easily incorporated into the second-level optimization problem.

Interpretation of the Optimal Solution

Equations 20, 21, 25, and 26 clearly show that exchange rates, corporate income tax, and customs duty influence the optimal sizes of purchasing and production lot. Exchange rates may have only a moderate impact on lot sizes since the exchange rate parameters that are present in both the numerator and denominator of the lot-sizing equations have the same functional forms. However, the cost terms in the objective function in Eq. 34 are proportional to the exchange rates.

To identify the impact of corporate income tax on lot-sizing equations, consider two extreme situations : (1) when depreciation is present and (2) when depreciation is over. Usually, depreciation terms a_k^{jr}, a_i^r, and b^{jr} are much bigger than all the other cost terms in Eqs. 25 and 26. Consider the case of $|R| = 1$ for simplicity. For situation (1), $\Psi_k^{jr} \approx (1-\xi^r)[b^{jr}(1-x_k^j) + a_k^{jr}]$ and the cycle time equation becomes $w_k^j \approx \sqrt{A_k^{jr}/(D_k^j[b^{jr}(1-x_k^j) + a_k^{jr}])}$; for situation, (2) a_k^{jr}, a_i^r, and $b^{jr} = 0$ and the cycle time equation becomes $w_k^j \approx \sqrt{(1-\xi^r)}\sqrt{A_k^{jr}/(D_k^j\Psi_k^j)}$. The cycle time equation for situation (1) is the same as the cycle time equation in the absence of corporate income tax ($\xi^r = 0$); however, that for situation (2) is $\sqrt{(1-\xi^r)}$ multiplied by the cycle time equation in the absence of corporate income tax. This analysis holds for the process cycle time (Eq. 21) as well as the purchase cycle time. Note that $\xi^r \approx 0.35$ has been recommended when investigating the feasibility of a plant design.[2] This means that the optimal lot size based on the profit after tax can be smaller than the optimal lot size based on the profit before tax by $\sqrt{(1-0.35)} \approx 0.8$ for a mature process where depreciation is over. Note that Eq. 32 and $^*\overline{\overline{V^j}} = 2^*\overline{V^j}$ indicate that the optimal material storage capacity is proportional to lot size, and so reducing the optimal lot size directly reduces the optimal storage size. The cost term of process i in the objective function in Eq. 34 is also $\sqrt{(1-\xi^r)}$ multiplied by the cost term in the absence of corporate income tax. This analysis clearly indicates that the optimization result without considering corporate income tax can be significantly erroneous, which is very common in most scheduling optimization models used in practical applications and academic research. Equation 26 indicates that customs duty has only a small effect on the optimal lot size.

The optimized total cost function in Eq. 34 is very useful for analyzing the profitability of MNCs under the condition of conforming with Eqs. 30 and 31 (i.e., no shortage in the material and currency inventories). Many important financial factors such as taxes, exchange rates, interest rates, and depreciations are already included in the equation. The cost terms in Eq. 34 are separable with respect to each process i, with the cost of process i being

$$2\sum_{i=1}^{|I|}\sqrt{\left(\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r\right)\left(\sum_{r=1}^{|R|}\chi^{r1}\Psi_i^r\right)D_i} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\sum_{i=1}^{|I|}\chi^{r1}(1-\xi^r)\pi_i^{jr}g_i^jD_i + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\xi^r\left[\sum_{i=1}^{|I|}P_i^{jr}(f_i^j-g_i^j)D_i - \sum_{r'\neq r}^{|R|}\sum_{i\in I_r}^{|I|}\chi^{r'r}P_i^{jr'}(f_i^j-g_i^j)D_i\right] + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\sum_{i\in\{D_i\}^+}^{|I|}0.5(1-{}'x_I)A_i^r \tag{39}$$

The current optimization model presented here is a single-period, deterministic model, but it could be extended in two directions: multiperiod formulation and uncertainty in exchange rates. Multiperiod formulation is required to cope with long-term variations of variables and parameters. Short-term random variations in exchange rates are an important research issue in operations research.[4–6] We succeeded in developing analytical methods for determining the optimal lot sizes of uncertain material flows using a PSW model and a graphical method.[11] Extending the current model with multiple periods and stochastic behavior represents a challenging future investigation.

Examples of Multinational Plant Design

Let us consider the example of a company in Nation C that is planning to move the up-stream processes of production facilities to Nation A or B so as to increase profitability. The superstructure of process flows are shown in Figure 5a, in which squares represent processes and circles represent storage units. Note that Figure 5 is a state task network, in which squares represent tasks and circles represent states. Processes A1 and B1 purchase raw materials, processes A2 and B2 are production units, and processes A3 and B3 transport materials from Nation A and B to Nation C. A rigorous profitability analysis first requires the second-level problem to be solved; however, solving a nonconvex mixed integer nonlinear programming formulation is nontrivial. We assume that the material and currency supply routes are already determined, and so the constraints on material and currency availability in Eqs. 30 and 31 are no longer necessary. Then, computing Eq. 34 immediately yields an economic comparison result of investing in the two nations. Table 3 shows the result of a spreadsheet computation for the substructure of this example. According to the economic comparison computation by using Eq. 34, investing nation B is more profitable than nation A. The spreadsheet equation in cell C56 is

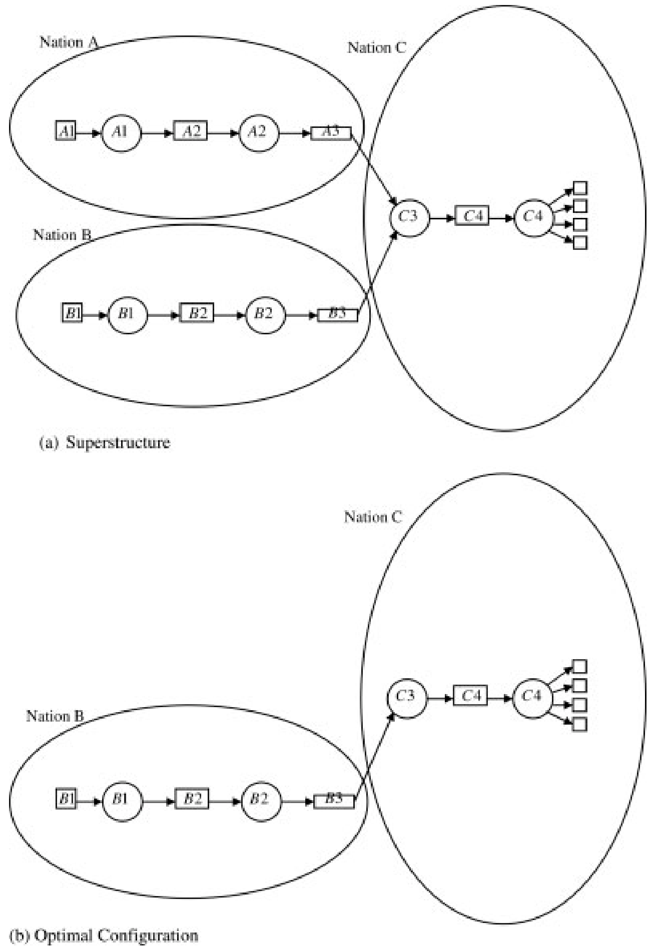

Figure 5. Example multi-national corporation. (a) Superstructure (b) Optimal Configuration.

$$\begin{aligned}C56 = {} & C6^*(1-C4)^*C20^*C9 - C6^*(1-C4)^*C36^*C40 \\ & + 2^*SQRT(C6^*(1-C4)^*C11^*C6^*((1-C4)^*C10 \\ & + 0.5^*((1-C4+C5)^*C18+C19 \\ & + C5^*(C12+C20))^*(1-C14)+(1-C4)^*C17)^*C9) \\ & + 2^*SQRT(C6^*(1-C4)^*C26^*C6^*((1-C4)^*C25 \\ & + (1-C29)^*(0.5^*((1-C4+C5)^*C18+C19) \\ & + (1-C4)^*C17)+(1-C30)^*(0.5^*((1-C4+C5)^*C34 \\ & + C35+C5^*C27)+(1-C4)^*C33))^*C24)\end{aligned}$$

Table 3. Profitability Analysis of Multinational Investment

A	B	C	D	E
2		Nation A	Nation B	Nation C
4	Corporate tax rate	0.28	0.33	0.4
5	Interest rate	0.05	0.12	0.02
6	Exchange rate	0.001	0.01	1
7				
8	Process 1			
9	Flow rate	100	100	
10	Capital cost			
11	Setup cost	10,000	500	
12	Variable cost	2000	100	
13	Optimal size	3.12471	2.45753	
14	SOTF	0.1	0.1	
15	Storage 1			
16	Flow rate	100	100	
17	Capital cost	100,000	8,000	
18	Operating cost	1000	80	
19	Opportunity cost	500	100	
20	Price	50,000	2000	
21	Optimal size	5.33525	4.74423	
22				
23	Process 2			
24	Flow rate	100	100	
25	Capital cost	1,000,000	80,000	
26	Setup cost	100,000	8,000	
27	Variable cost	3,000	200	
28	Optimal size	2.80334	2.81384	
29	SOTF (feedstock)	0.1	0.1	
30	SOTF (product)	0.1	0.1	
31	Storage 2			
32	Flow rate	100	100	
33	Capital cost	200,000	15,000	
34	Operating cost	2,000	150	
35	Opportunity cost	1,000	200	
36	Price	1,000,000	90,000	
37	Optimal size	69.204	82.1212	
38				
39	Process 3			
40	Flow rate	100	100	
41	Capital cost			
42	Setup cost	1E + 07	1,100,000	
43	Variable cost	10,000	1,000	
44	Optimal size	74.09	88.432	
45	Customs duty rate	0.04	0.04	
46	SOTF (feedstock)	0.1	0.1	
47	SOTF (product)	0.1	0.1	
48	Storage 3			
49	Flow rate			100
50	Capital cost			300
51	Operating cost			3
52	Opportunity cost			1
53	Price			1,200
54				
55				
56	Optimal total cost	−21636	−22298	

$$+ 2^*\mathrm{SQRT}(C6^*(1 - C4)^*C42^*(C6^*(1 - C4)^*C41 + C6^*(1 - C46)^*(0.5^*((1 - C4 + C5)^*C34 + C35) + (1 - C4)^*C33) + E6^*(1 - C47)^*(0.5^*((1 - C4 + C5)^*E51 + E52 + C5^*(C43 + C45)) + (1 - C4)^*E50))^*C40) + C6^*(C12 + C27 + C43)^*C40^*(1 - C4) + C6^*C5^*0.5^*((1 - C46)^*C42 + (1 - C29)^*C26 + (1 - C14)^*C11)$$

Figure 5b shows the optimal configuration of the MNC. The optimal batch sizes of A1 and B1 which were C13 and D13 respectively in Table 3 were computed by using Eq. 20.

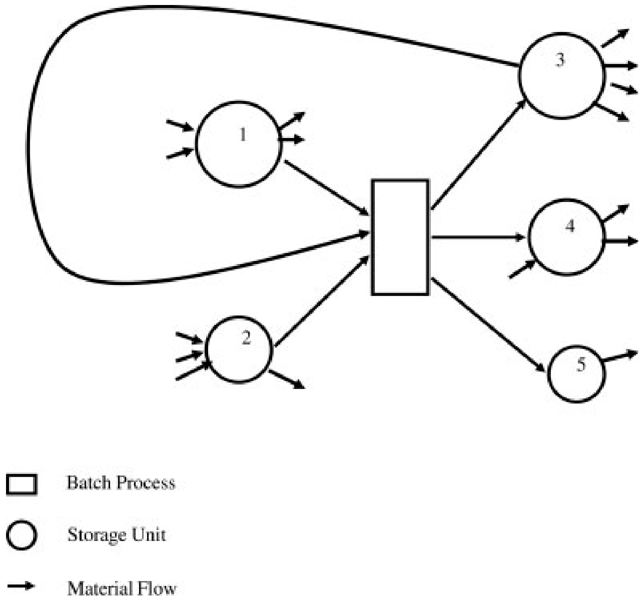

Figure 6. Optimal size of batch process connected by storage units.

The optimal batch sizes of A2, B2, A3, and B3 which were C28, D28, C44, and D44 respectively in Table 3 were computed by using Eq. 21. The optimal storage sizes of A1, B1, A2, and B2 which were C21, D21, C37, and D37 respectively in Table 3 were computed by using $^*\overline{\overline{V^j}} = 2^*\overline{V^j}$ and Eq. 32. Note that the prices of the materials in storage units A2 and B2 correspond to the transfer price. The operating cost in Table 3 means the annual inventory operating cost (h^{jr}). The opportunity cost in Table 3 means the opportunity cost of inventory holding (γ^{jr}).

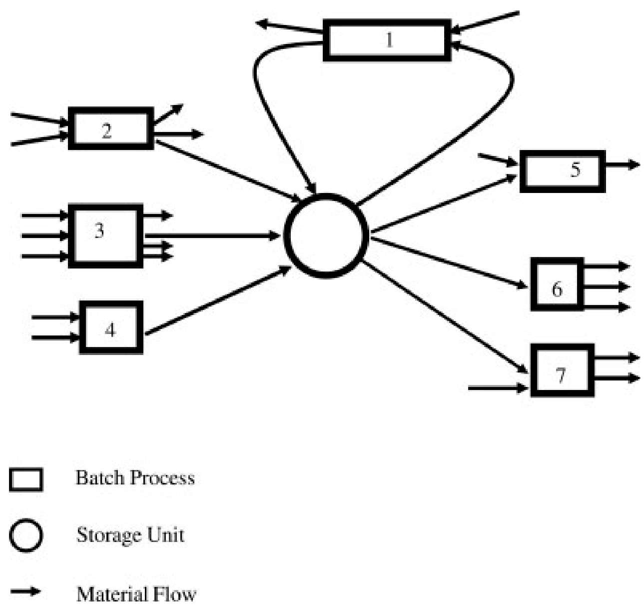

Figure 7. Optimal size of storage unit connected by multiple supply and consumption processes.

Table 4. Optimal Batch Size in Batch-Storage Network

A	B	C	D	E	F	G	H
2	Process						
3	Nationality	Nation A					
4	Flow rate	100					
5	Capital cost	1000000					
6	Setup cost	500000					
7	Corporate tax rate	0.28					
8	Exchange rate	0.001					
9	SOTF (feedstock)	0.1					
10	SOTF (product)	0.2					
11							
12	Storage Units	Storage 1	Storage 2	Storage 3	Storage 3	Storage 4	Storage 5
13							
14		Feedstock	Feedstock	Feedstock	Product	Product	Product
15	Nationality	Nation A	Nation A	Nation A	Nation A	Nation A	Nation B
16	Flow rate	70	10	20	25	45	30
17	Capital cost	100000	100000	100000	100000	100000	5000
18	Operating cost	1000	1000	1000	1000	1000	50
19	Opportunity cost	500	500	500	500	500	100
20	Price						2000
21	Customs duty rate						0.04
22	Interest rate	0.05	0.05	0.05	0.05	0.05	0.1
23	Exchange rate	0.001	0.001	0.001	0.001	0.001	0.01
24	Corporate tax rate	0.28	0.28	0.28	0.28	0.28	0.33
25							
26	Optimal batch size =	6.569046					
27	Cost of process =	5500.248					

The application of the results of this study is not limited to a simple serial system, as in Figure 5, but can also be applied to a BSN of arbitrary complexity that is spread over many nations. The design problem of a BSN can be decomposed into the design problem of processes, as shown at Figure 6, and the design problem of storage units, as shown at Figure 7. Processes in such a BSN are connected by many feedstock or product storage units, as shown in Figure 6, in which squares represent processes and circles represent storage units. Note that some materials may be recycled, such as that in storage unit 3. Then, the optimal batch size (using Eq. 21) and the cost of process (using Eq. 39) can be computed as long as the average flow rate is known a priori. The average flow rates are determined by solving second-level optimization problem. Note that storage unit 5 is located in a different nation, which is possible when transportation is included in the model of the process. Table 4 shows the result of a spreadsheet computation; the spreadsheet equation used to compute the optimal batch size in cell C26 is

$$\begin{aligned} C26 = {} & SQRT(C8^{*}(1-C7^{*}C6^{*}C4/(C8^{*}(1-C7)^{*}C5 \\ & +(1-C9)^{*}(C23^{*}(C16/C4)^{*}(0.5^{*}((1-C24+C22)^{*}C18 \\ & +C19)+(1-C24)^{*}C17)+D23^{*}(D16/C4)^{*} \\ & (0.5^{*}((1-D24+D22)^{*}D18+D19)+(1-D24)^{*}D17)+ \\ & E23^{*}(E16/C4)^{*}(0.5^{*}((1-E24+E22)^{*}E18 \\ & +E19)+(1-E24)^{*}E17))+(1-C10)^{*}(F23^{*}(F16/C4)^{*} \\ & (0.5^{*}((1-F24+F22)^{*}F18+F19+F22^{*}F21) \\ & +(1-F24)^{*}F17)+G23^{*}(G16/C4)^{*}(0.5^{*}((1-G24 \\ & +G22)^{*}G18+G19+G22^{*}G21)+(1-G24)^{*}G17) \\ & +H23^{*}(H16/C4)^{*}(0.5^{*}((1-H24+H22)^{*}H18+H19 \\ & +H22^{*}H21)+(1-H24)^{*}H17)))) \end{aligned}$$

The operating cost in Table 4 means the annual inventory operating cost (h^{jr}). The opportunity cost in Table 4 means the opportunity cost of inventory holding (γ^{jr}).

A storage unit in the BSN of arbitrary complexity is connected by many supplying and consumption processes, as shown in Figure 7, in which squares represent processes and circles represent storage units. At first, the batch sizes connected to the storage unit should be computed by using Eq. 21 or the above procedure. The optimal storage size can be

Table 5. Optimal Storage Size in Batch-Storage Network

A	B	C	D	E
2		Batch size	FC or PY	SOTF
4	Process 1	500	0.2	0.12
5	Process 1	200	0.1	0.12
6	Process 2	600	0.7	0.07
7	Process 3	400	0.5	0.21
8	Process 4	900	1	0.17
9	Process 5	100	0.4	0.13
10	Process 6	800	0.8	0.15
11	Process 7	300	1	0.24
12				
13	SOTF = Storage operation time fraction			
14	FC or PY = Feedstock composition or product yield			
15				
16				
17	Optimal storage size =		2208	

computed using $^*\overline{\overline{V^j}} = 2^*\overline{V^j}$ and Eq. 32 if the connected batch sizes are already determined. Table 5 shows the spreadsheet computation result; the spreadsheet equation in cell D17 is

$$\begin{aligned} \mathrm{D17} = \mathrm{C4^*D4^*(1-E4) + C5^*D5^*(1-E5) + C6^*D6^*(1-E6)} \\ \mathrm{+ C7^*D7^*(1-E7) + C8^*D8^*(1-E8) + C9^*D9^*(1-E9)} \\ \mathrm{+ C10^*D10^*(1-E10) + C11^*D11^*(1-E11)} \end{aligned}$$

Conclusions

This article has proposed a mathematical optimization framework in which the material and currency flows of MNCs are considered simultaneously, whilst focusing on the impact of macroscopic economic factors such as exchange rates and taxes on lot sizing and timing decisions of supply chain operations. The optimal production plan in the presence of binding multiple currency financial constraints differed from the plan generated under the assumption of a single currency. The inclusion of corporate income tax in the model decreased the optimal production lot and storage sizes, typically by 20%. The model covered various cost factors such as the opportunity costs of annualized capital investment and currency/material inventories, the prices of raw material purchases and final product selling, processing, and financial transaction setup, interest rates, transfer prices, taxes, and depreciations. Lot sizes, storage sizes, and startup times of material processing were determined by analytical equations. In spite of the enlarged scope of the problem, the computational burden was light due to the use of mostly analytical methods and the numerical simplicity of the subproblem structure, such as separable concave minimization.[1] The BSN used in this study is very general, and can cover most business activities of MNCs such as raw material procurement, production, inventory management, distribution, and transportation, as well as financial transactions. This study will contribute to the optimal global management of material and currency flows, and lead to the genuine optimization of the operation of a multinational enterprise.

Acknowledgments

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (KRF-2006-311-D00382).

Notation

a_k^{jr} = annualized capital cost of raw material purchasing facility, currency/L/year
a_i^r = annualized capital cost of unit i paid by currency r, currency/L/year
b^{jr} = annualized capital cost of storage facuility j paid by currency r, currency/L/year
A_k^{jr} = setup cost of feedstock materials, currency/batch
A_i^r = setup cost of noncontinuous units, currency/batch
A_n^r = setup cost of financial investment, currency/transaction
A_l^r = setup cost of bank loan, currency/transaction
$A^{rr'}$ = setup cost of currency transfer, currency/transaction
B_k^j = raw material order size, L/batch
B_i = noncontinuous unit size, L/batch
B_m^j = final product delivery size, L/batch
$C^r(0)$ = initial cash inventory of currency r
$C^r(t)$ = cash inventory of currency r at present time t
$\overline{C}^r$ = average level of currency inventory
$\overline{\overline{C}}^r$ = upper level of currency inventory
$\underline{\underline{C}}^r$ = lower level of currency inventory
D_k^j = average material flow of raw material supply, L/year
$\{D_k^j\}^+$ = The set of index k that has positive value of D_k^j
D_m^j = average material flow of customer demand, L/year
D_i = average material flow through noncontinuous units, L/year
$\{D_i\}^+$ = The set of index i that has positive value of D_i
E_l^r = average currency flow rate of bank loan l, currency/year
$\{E_l^r\}^+$ = The set of index l that has positive value of E_l^r
E_n^r = average currency flow rate of financial investment n, currency/year
$\{E_n^r\}^+$ = The set of index n that has positive value of E_n^r
E_o^r = average currency flow rate of dividend to stockholders o, currency/year
$E_{\bar{o}}^r$ = average currency flow rate of corporate income tax, currency/year
$E^{rr'}$ = average currency flow rate of currency transfer from r to r', currency/year
$\{E^{rr'}\}^+$ = The set of index r, r' that has positive value of $\mathrm{E}^{rr'}$
$\hat{E}^r$ = average cash flow rate of sales tax paid by currency r, currency/year
$F_i(t)$ = periodic square wave flow
f_i^j = feedstock composition of unit i
g_i^j = product yield of unit i
H^{jr} = annual inventory holding cost, currency/L/year
h^{jr} = annual inventory operating cost, currency/L/year
I = noncontinuous process set
I_r = noncontinuous process subset owned by the nation that uses currency r
J = material strorage set
J_r = material strorage subset owned by the nation that uses currency r
J_s = material strorage subset owned by subsidiary s
K(j) = raw material supplier set for stroage j
M(j) = consumer set for stroage j
P_k^{jr} = price of raw material j from supplier k paid by currency r, currency/L
P_m^{jr} = sales price of finished products to customer m paid by currency r, currency/L
PSW = the first type of periodic square wave function defined by Eq. 1
PSW' = the second type of periodic square wave function defined by Eq. 2
$\overline{PSW}$ = average level of the first type of periodic square wave function defined by Table 1
$\overline{PSW'}$ = average level of the second type of periodic square wave function defined by Table 1
$\overline{\overline{PSW}}$ = upper bound of the first type of periodic square wave function defined by Table 1
$\overline{\overline{PSW'}}$ = upper bound of the second type of periodic square wave function defined by Table 1
$\underline{\underline{PSW}}$ = lower bound of the first type of periodic square wave function defined by Table 1
$\underline{\underline{PSW'}}$ = lower bound of the second type of periodic square wave function defined by Table 1
R = currency set
R_s = currency subset used by subsidiary s
S = subsidiary set
t_m^j = startup time of customer demand, year
$'t_i$ = startup time of feedstock feeding to noncontinus unit i, year
t_i' = startup time of product discharing form noncontinuous unit i, year
t_k^j = startup time of raw material purchasing, year
t_c^r = minimum of the collecton times of account receivables, year

t_l^r = startup time of bank loan l, year
t_n^r = startup time of financial investment n, year
t_o^r = startup time of dividend to stockholders, year
$\hat{t}_m^{jr}$ = startup time of sales tax, year
$t^{rr'}$ = startup time of currency transfer, year
Δt_k^j = disbursement drifting time of account payables, year
Δt_m^j = collection drifting time of account receivables, year
Δt_l^r = financial investment period, year
Δt_n^r = financial investment period, year
$\overline{\overline{V^j}}$ = upper bound of inventory hold-up, L
$\underline{\underline{V^j}}$ = lower bound of inventory hold-up, L
$V^j(t)$ = inventory hold-up, L
$V^j(0)$ = initial inventory hold-up, L
$\overline{V}^j$ = time average inventory hold-up, L
x_k^j = storage operation time fraction of purchasing raw materials
x_i^j = storage operation time fraction of feeding to noncontinuous unit i
x_i^{j} = storage operation time fraction of discharing form noncontinuous unit i
x_m^j = storage operation time fraction of customer demand

Greek letters

ς^r = sales tax rate paid by currency r, currency/currency
ξ^r = Corporate income tax rate paid by currency r, currency/currency
$\chi^{rr'}$ = foreign currency exchange rate from r to r', currency r'/currency r
χ^{r1} = foreign currency exchange rate from r to 1, numeraire currency/currency r
γ^{jr} = opportunity cost of inventory holding paid by currency r, currency/L/year
η^r = opportunity cost of cash holding paid by currency r, currency/currency/year
κ_l^r = interest rate of bank loan l paid by currency r, currency/currency/year
κ_n^r = interest rate of financial investment n paid by currency r, currency/currency/year
π_i^{jr} = customs duty rate of material in storage j transported by process i, paid by currency r, currency/L
ω_m^j = cycle time of customer demand, year
ω_k^j = cycle time of raw material purchasing, year
ω_i = cycle time of noncontinuous units, year
ω_l^r = cycle time of bank loan l, year
ω_n^r = cycle time of financial investment n, year
ω_o^r = cycle time of dividend to stockholders, year
ω^r = cycle time of sales tax, year
$\omega^{rr'}$ = cycle time of currency transfer from r to r', year
Ψ_i^r = aggregated cost for process i defined by Eq. 26, currency/L/year
Ψ_k^{jr} = aggregated cost for raw material purchase defined by Eq. 25, currency/L/year
Ψ_l^r = aggregated cost for bank loan l defined by Eq. 28, currency/currency/year
Ψ_n^r = aggregated cost for financial investment n defined by Eq. 27, currency/currency/year
$\Psi^{rr'}$ = aggregated cost for financial investment n defined by Eq. 29, currency/currency/year

Subscripts

i = noncontinuous unit index
k = index of raw material vendors
m = index of finished product customers
l = index of bank loan
n = index of financial investments
o = index of stockholders
$\overline{o}$ = index of corporate income tax
s = index of subsidiary

Superscripts

j = storage index
r = index of currency in nation

Special functions

int[.] = truncation function to make integer
res[.] = truncation function to make integer
$|X|$ = Number of elements in set X

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Appendix

Appendix A: The Solution of the Kuhn-Tucker Conditions of the First Problem

Because the constraints are $\underline{\underline{V^j}} \geq 0$ and $\underline{\underline{C}} \geq 0$, the Lagrangian is:

$$L = TC - \sum_{j=1}^{|J|} \underline{\underline{\lambda}}^j [\underline{\underline{V^j}}] - \sum_{r=1}^{|R|} \underline{\underline{\lambda}}^r [\underline{\underline{C^r}}] \tag{A1}$$

where $\underline{\underline{\lambda}}^j$ and $\underline{\underline{\lambda}}$ are nonnegative Lagrange multipliers. $\underline{\underline{V^j}}$ and $\underline{\underline{C}}$ are computed from Eqs. 7 and 16. For simplicity, we do not explicitly include the nonnegativity constraints of the design variables. The Kuhn-Tucker conditions of Eq. A1 are:

$$\begin{aligned}\frac{\partial L}{\partial t_k^j} &= \sum_{r=1}^{|R|} \chi^{r1} \left[\left((1-\xi^r) h^{jr} + \gamma^{jr} \right) \frac{\partial \overline{V^j}}{\partial t_k^j} + (1-\xi^r) b^{jr} \frac{\partial \overline{\overline{V^j}}}{\partial t_k^j} \right] \\ &\quad - \underline{\underline{\lambda}}^j \frac{\partial \underline{\underline{V^j}}}{\partial t_k^j} - \sum_{r=1}^{|R|} \left[\underline{\underline{\lambda}}^r \frac{\partial \underline{\underline{C}}^r}{\partial t_k^j} - \chi^{r1} \eta^r \frac{\partial \overline{C}^r}{\partial t_k^j} \right] \\ &= - \sum_{r=1}^{|R|} \chi^{r1} \left((1-\xi^r) h^{jr} + \gamma^{jr} + (1-\xi^r) b^{jr} \right) D_k^j + \underline{\underline{\lambda}}^j D_k^j \\ &\quad - \sum_{r=1}^{|R|} \underline{\underline{\lambda}}^r \left[P_k^{jr} D_k^j + \frac{A_k^{jr}}{\omega_k^j} \right] + \sum_{r=1}^{|R|} \chi^{r1} \eta^r \left[P_k^{jr} D_k^j + \frac{A_k^{jr}}{\omega_k^j} \right] = 0\end{aligned} \tag{A2}$$

$$\frac{\partial L}{\partial \omega_k^j} = \sum_{r=1}^{|R|} \chi^{r1}\left[-\frac{(1-\xi^r)A_k^{jr}}{(\omega_k^j)^2} + (1-\xi^r)a_k^{jr}D_k^j\right] + \sum_{r=1}^{|R|} \chi^{r1}\left[\left((1-\xi^r)h^{jr}+\gamma^{jr}\right)\frac{\partial \overline{V^j}}{\partial \omega_k^j} + (1-\xi^r)b^{jr}\frac{\partial \overline{\overline{V^j}}}{\partial \omega_k^j}\right] - \underline{\underline{\lambda}}^j\frac{\partial \underline{\underline{V^j}}}{\partial \omega_k^j} + \sum_{r=1}^{|R|}\left[\chi^{r1}\eta^r\frac{\partial \overline{C}^r}{\partial \omega_k^j} - \underline{\underline{\lambda}}^r\frac{\partial \underline{\underline{C}}^r}{\partial \omega_k^j}\right]$$

$$= -\frac{\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_k^{jr}}{(\omega_k^j)^2} + \sum_{r=1}^{|R|}\chi^{r1}\left[\left(\frac{(1-\xi^r)h^{jr}+\gamma^{jr}}{2} + (1-\xi^r)b^{jr}\right)(1-x_k^j) + (1-\xi^r)a_k^{jr}\right]D_k^j - \sum_{r=1}^{|R|}\chi^{r1}\eta^r\left[\frac{(P_k^{jr})(1-x_k^j)}{2}\,D_k^j + \frac{A_k^{jr}t_k^j}{(\omega_k^j)^2} - \frac{\partial}{\partial \omega_k^j}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] + \sum_{r=1}^{|R|}\underline{\underline{\lambda}}^r\left[(0.5h^{jr}+P_k^{jr})(1-x_k^j)D_k^j + \frac{A_k^{jr}t_k^j}{(\omega_k^j)^2} - \frac{\partial}{\partial \omega_k^j}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] = 0 \qquad \text{(A3)}$$

$$\frac{\partial L}{\partial t_i} = \sum_{j=1}^{|J|}\left[\sum_{r=1}^{|R|}\chi^{r1}\left[\left((1-\xi^r)h^{jr}+\gamma^{jr}\right)\frac{\partial \overline{V^j}}{\partial' t_i} + (1-\xi^r)b^{jr}\frac{\partial \overline{\overline{V^j}}}{\partial' t_i}\right] - \underline{\underline{\lambda}}^j\frac{\partial \underline{\underline{V^j}}}{\partial' t_i}\right] + \sum_{r=1}^{|R|}\left[\chi^{r1}\eta^r\frac{\partial \overline{C}^r}{\partial' t_i} - \underline{\underline{\lambda}}^r\frac{\partial \underline{\underline{C}}^r}{\partial' t_i}\right]$$

$$= \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\left((1-\xi^r)h^{jr}+\gamma^{jr}+(1-\xi^r)b^{jr}\right)\left(f_i^j-g_i^j\right)D_i - \sum_{j=1}^{|J|}\underline{\underline{\lambda}}^j\left(f_i^j-g_i^j\right)D_i + \sum_{r=1}^{|R|}\chi^{r1}\eta^r\left[\frac{A_i^r}{\omega_i} + \sum_{j=1}^{|J|}\pi_i^{jr}g_i^jD_i\right] - \underline{\underline{\lambda}}^r\left[\frac{A_i^r}{\omega_i} + \sum_{j=1}^{|J|}\pi_i^{jr}g_i^jD_i\right] = 0 \qquad \text{(A4)}$$

$$\frac{\partial L}{\partial \omega_i} = -\frac{\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r}{(\omega_i)^2} + \sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)a_i^rD_i + \sum_{j=1}^{|J|}\left[\sum_{r=1}^{|R|}\chi^{r1}\left[\left((1-\xi^r)h^{jr}+\gamma^{jr}\right)\frac{\partial \overline{V^j}}{\partial \omega_i} + (1-\xi^r)b^{jr}\frac{\partial \overline{\overline{V^j}}}{\partial \omega_i}\right] - \underline{\underline{\lambda}}^j\frac{\partial \underline{\underline{V^j}}}{\partial \omega_i}\right] + \sum_{r=1}^{|R|}\left[\chi^{r1}\eta^r\frac{\partial \overline{C}^r}{\partial \omega_i} - \underline{\underline{\lambda}}^r\frac{\partial \underline{\underline{C}}^r}{\partial \omega_i}\right]$$

$$= -\frac{\sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)A_i^r}{(\omega_i)^2} + \sum_{r=1}^{|R|}\chi^{r1}(1-\xi^r)a_i^rD_i + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}\left((1-\xi^r)h^{jr}+\gamma^{jr}\right)\left\{-\frac{(1-x_i')}{2}f_i^jD_i + \frac{(1-x_i')}{2}g_i^jD_i - g_i^jD_i\frac{\partial \Delta t_i}{\partial \omega_i}\right\} + \sum_{r=1}^{|R|}\sum_{j=1}^{|J|}\chi^{r1}(1-\xi^r)b^{jr}\left\{(1-x_i')g_i^jD_i - g_i^jD_i\frac{\partial \Delta t_i}{\partial \omega_i}\right\}$$

$$- \sum_{j=1}^{|J|}\underline{\underline{\lambda}}^j\left\{-(1-x_l')f_i^jD_i - g_i^jD_i\frac{\partial \Delta t_i}{\partial \omega_i}\right\} - \sum_{r=1}^{|R|}\chi^{r1}\eta^r\left[\frac{A_i^rt_i'}{(\omega_i)^2} - \frac{\partial}{\partial \omega_i}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right) + 0.5\sum_{j=1}^{|J|}\pi_i^{jr}g_i^jD_i(1-x_i')\right] + \sum_{r=1}^{|R|}\underline{\underline{\lambda}}^r\left[\sum_{j=1}^{|J|}h^{jr}\left[\frac{(1-x_l')}{2}f_i^jD_i + \frac{(1-x_i')}{2}g_i^jD_i\right] + \frac{A_i^rt_l'}{(\omega_i)^2}\right] + \sum_{r=1}^{|R|}\underline{\underline{\lambda}}^r\left[\sum_{j=1}^{|J|}\pi_i^{jr}g_i^jD_i(1-x_i') - \frac{\partial}{\partial \omega_i}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] = 0 \qquad \text{(A5)}$$

$$\frac{\partial L}{\partial t_n^r} = \chi^{r1}\eta^r\frac{\partial \overline{C}^r}{\partial t_n^r} - \underline{\underline{\lambda}}^r\frac{\partial \underline{\underline{C}}}{\partial t_n^r} = \chi^{r1}\eta^r\left[-\Delta t_n^r\kappa_n^rE_n^r + \frac{A_n^r}{\omega_n^r} + \frac{\partial}{\partial t_n^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] - \underline{\underline{\lambda}}^r\left[-\Delta t_n^r\kappa_n^rE_n^r + \frac{A_n^r}{\omega_n^r} + \frac{\partial}{\partial t_n^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] = 0 \qquad \text{(A6)}$$

$$\frac{\partial L}{\partial \omega_n^r} = -\frac{\chi^{r1}(1-\xi^r)A_n^r}{(\omega_n^r)^2} + \chi^{r1}\eta\frac{\partial \overline{C}}{\partial \omega_n^r} - \underline{\underline{\lambda}}\frac{\partial \underline{\underline{C}}}{\partial \omega_n^r} = -\frac{\chi^{r1}(1-\xi^r)A_n^r}{(\omega_n^r)^2} + \chi^{r1}\eta^r\left[0.5E_n^r\kappa_n^r\Delta t_n^r - \frac{A_n^rt_n^r}{(\omega_n^r)^2} + \frac{\partial}{\partial \omega_n^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] - \underline{\underline{\lambda}}^r\left[-E_n^r - \frac{A_n^rt_n^r}{(\omega_n^r)^2} + \frac{\partial}{\partial \omega_n^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] = 0 \qquad \text{(A7)}$$

$$\frac{\partial L}{\partial t_l^r} = \chi^{r1}\eta^r\frac{\partial \overline{C}^r}{\partial t_l^r} - \underline{\underline{\lambda}}^r\frac{\partial \underline{\underline{C}}}{\partial t_l^r} = \chi^{r1}\eta^r\left[\Delta t_l^r\kappa_l^rE_l^r + \frac{A_l^r}{\omega_l^r} + \frac{\partial}{\partial t_l^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] - \underline{\underline{\lambda}}^r\left[\Delta t_l^r\kappa_l^rE_l^r + \frac{A_l^r}{\omega_l^r} + \frac{\partial}{\partial t_l^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|O|}E_o^r\right)\right] = 0 \qquad \text{(A8)}$$

$$\frac{\partial L}{\partial \omega_l^r} = -\frac{\chi^{r1}(1-\xi^r)A_l^r}{(\omega_l^r)^2} + \chi^{r1}\eta\frac{\partial \overline{C}}{\partial \omega_l^r} - \underline{\underline{\lambda}}^r\frac{\partial \underline{\underline{C}}}{\partial \omega_l^r} = -\frac{\chi^{r1}(1-\xi^r)A_l^r}{(\omega_l^r)^2} + \chi^{r1}\eta^r\left[-0.5E_l^r\kappa_l^r\Delta t_l^r - \frac{A_l^rt_l^r}{(\omega_l^r)^2} + \frac{\partial}{\partial \omega_l^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|o|}E_o^r\right)\right] - \underline{\underline{\lambda}}^r\left[-(1+\kappa_l^r\Delta t_l^r)E_l^r - \frac{A_l^rt_l^r}{(\omega_l^r)^2} + \frac{\partial}{\partial \omega_l^r}\left(t_{\bar{o}}^r\sum_{o=1}^{|o|}E_o^r\right)\right] = 0 \qquad \text{(A9)}$$

$$\frac{\partial L}{\partial t^{rr'}} = \frac{\partial}{\partial t^{rr'}} \sum_{r=1}^{|R|} \chi^{r1} \eta^r \overline{C}^r - \frac{\partial}{\partial t^{rr'}} \sum_{r=1}^{|R|} \underline{\underline{\lambda}}^r \underline{\underline{C}} = \frac{\partial}{\partial t^{rr'}} \sum_{r=1}^{|R|} \chi^{r1} \eta^r \times \left[\sum_{r' \neq r}^{|R|} \chi^{r'r} [-D^{r'r} t^{r'r}] - \sum_{r' \neq r}^{|R|} [-D^{rr'} t^{rr'}] - \sum_{r' \neq r}^{|R|} A^{r'r} \left[-\frac{t^{r'r}}{\omega^{r'r}} \right] \right] - \frac{\partial}{\partial t^{rr'}} \sum_{r=1}^{|R|} \underline{\underline{\lambda}}^r \left[-\sum_{r' \neq r}^{|R|} \chi^{r'r} E^{r'r} t^{r'r} - \sum_{r' \neq r}^{|R|} [-E^{rr'} t^{rr'}] - \sum_{r' \neq r}^{|R|} A^{r'r} \left[-\frac{t^{r'r}}{\omega^{r'r}} \right] \right] = \left[-\chi^{r'1} \eta^{r'} \chi^{rr'} E^{rr'} + \chi^{r1} \eta^r E^{rr'} + \frac{\chi^{r'1} \eta^{r'} A^{rr'}}{\omega^{rr'}} \right] - \left[-\underline{\underline{\lambda}}^{r'} \chi^{rr'} E^{rr'} + \underline{\underline{\lambda}}^r E^{rr'} + \frac{\underline{\underline{\lambda}}^{r'} A^{rr'}}{\omega^{rr'}} \right] = 0 \tag{A10}$$

$$\frac{\partial L}{\partial \omega^{rr'}} = \frac{\partial}{\partial \omega^{rr'}} \sum_{r=1}^{|R|} \chi^{r1} \eta^r \overline{C}^r - \frac{\partial}{\partial \omega^{rr'}} \sum_{r=1}^{|R|} \underline{\underline{\lambda}}^r \underline{\underline{C}} + \frac{\partial}{\partial \omega^{rr'}} \sum_{r=1}^{|R|} \sum_{r' \neq r}^{|R|} \chi^{r'1} \frac{(1-\xi^{r'}) A^{rr'}}{\omega^{rr'}} = \frac{\partial}{\partial \omega^{rr'}} \sum_{r=1}^{|R|} \chi^{r1} \eta^r \times \left[\sum_{r' \neq r}^{|R|} \chi^{r'r} [0.5 E^{r'r} \omega^{r'r}] - \sum_{r' \neq r}^{|R|} [0.5 E^{rr'} \omega^{rr'}] - \sum_{r' \neq r}^{|R|} A^{r'r} \left[-\frac{t^{r'r}}{\omega^{r'r}} \right] \right] - \frac{\partial}{\partial \omega^{rr'}} \sum_{r=1}^{|R|} \underline{\underline{\lambda}}^r \left[-\sum_{r' \neq r}^{|R|} [E^{rr'} \omega^{rr'}] - \sum_{r' \neq r}^{|R|} A^{r'r} \left[-\frac{t^{r'r}}{\omega^{r'r}} \right] \right] + \chi^{r'1} \left[-\frac{(1-\xi^{r'}) A^{rr'}}{(\omega^{rr'})^2} \right] = \left[\chi^{r'1} \eta^{r'} \chi^{rr'} 0.5 E^{rr'} - 0.5 \chi^{r1} \eta^r E^{rr'} - \chi^{r'1} \eta^{r'} A^{rr'} \frac{t^{rr'}}{(\omega^{rr'})^2} \right] - \left[-\underline{\underline{\lambda}}^r E^{rr'} - \underline{\underline{\lambda}}^{r'} A^{rr'} \frac{t^{rr'}}{(\omega^{rr'})^2} \right] + \chi^{r'1} \left[-\frac{(1-\xi^{r'}) A^{rr'}}{(\omega^{rr'})^2} + a^{rr'} E^{rr'} \right] = 0 \tag{A11}$$

$$\underline{\underline{\lambda}}^j \underline{\underline{V}}^j = 0 \text{ and } \underline{\underline{\lambda}}^r \underline{\underline{C}}^r = 0 \tag{A12}$$

Solving Eqs. A2, A4, A6, A8, and A10 gives the values of multipliers:

$$\underline{\underline{\lambda}}^j = \sum_{r=1}^{|R|} \chi^{r1} \left((1-\xi^r) h^{jr} + \gamma^{jr} + (1-\xi^r) b^{jr} \right), \ \underline{\underline{\lambda}}^r = \chi^{r1} \eta^r \tag{A13}$$

Solving Eqs. A3, A5, A7, A9, and A11 with Eq. A13 gives the optimal values of the cycle times in the main text.

Manuscript received Sept. 17, 2006, and revision received Feb. 13, 2007.